1) (16 points) Let the reduced echelon form of a matrix $A = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
-1 & 2 & 0 & 0 \\
2 & -3 & 0 & 0
\end{bmatrix}$

a) Is $A$ invertible? Explain.
b) Do the columns of $A$ form a basis for $\mathbb{R}^4$? Explain.
c) Find bases and dimensions of $\text{Nul}(A)$ and $\text{Row}(A)$.
d) Is $\lambda = 0$ an eigenvalue of $A$? Explain.

2) (14 points) Let $L : \mathbb{R}^m \to \mathbb{R}^n$ be a linear map.
a) What’s the size of the standard matrix $A$ of $L$?
b) Find the relationship between $m$ and $n$ if
i) $L$ is one-to-one but not onto; ii) $L$ is onto but not one-to-one; iii) $L$ is one-to-one and onto. Explain.

3) (15 points) Let $A$ be an $4\times4$ matrix with $\det A = 4$ and diagonalizable, i.e., $A = PDP^{-1}$.
a) Compute $\det(2A^2A^T)$ and $\det D^3$.
b) Can $A$ have an eigenvalue of multiplicity two and the corresponding eigenspace of dimension one? Explain.

4) (20 points) Let $A = \begin{bmatrix}
1 & 1 & 0 \\
1 & 2 & 1 \\
0 & 1 & 1
\end{bmatrix}$.
a) Find the eigenvalues of $A$ and the corresponding eigenvectors.
b) Find the orthogonal diagonalization of $A$.

5) (20 points) Let $V = \text{span} \{ (1, 1, 1, 1)^T, (1, 0, 1, 1)^T, (0, 0, 1, 2)^T \}$.
a) Show that the dim $V = 3$.
b) Use Gram-Schmidt to find an orthonormal basis for $V$.
c) Find a QR factorization for $A$.

6) (15 points) a) Find the quadratic form whose standard matrix is $A$ in problem 4).
b) Is $Q$ positive definite? negative definite? or indefinite? Explain.
c) Find the change of variable that eliminates the mixed product terms from $Q$. 

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