Math 337 ——Final Exam —-Fall 2018

1) (16 points) Let the reduced echelon form of a matrix A=$[(1,0,0,0)^T(0,1,0,0)^T(-1,2,0,0)^T(2,-3,0,0)^T]$

a) Is A invertible? Explain.

b) Do the columns of A form a basis for R^4 ? Explain.

c) Find bases and dimensions of Nul(A) and Row(A).

d) Is $\lambda = 0$ and eigenvalue of A? Explain.

2) (14 points) Let $L : R^m \to R^n$ be a linear map.

a) What's the size of the standard matrix A of L?

b) Find the relationship between m and n if

i) L is one-to-one but not onto; ii) L is onto but not one-to-one; iii) L is one-to-one and onto. Explain.

3) (15 points) Let A be an 4x4 matrix with detA=4 and diagonalizable, i.e., $A = PDP^{-1}$.

a) Compute $\det(2A^2A^T)$ and det D^3.

b) Can A have an eigenvalue of multiplicity two and the corresponding eigenspace of dimension one? Explain.

4) (20 points) Let $A = [(1,1,0)^T((1,2,1)^T(0,1,1)^T]$.

a) Find the eigenvalues of A and the corresponding eigenvectors.

b) Find the orthogonal diagonalization of A.

5) (20 points) Let V=span $\{(1,1,1,1)^T(1,0,1,1)^T(0,0,1,2)^T\}$.

a) Show that the dim V=3.

b) Use Gram-Schmidt to find an orthonormal basis for V.

c) Find a QR factorization for A.

6) (15 points) a) Find the quadratic form whose standard matrix is A in problem 4).

b) Is Q positive definite? negative definite? or indefinite? Explain.

c) Find the change of variable that eliminates the mixed product terms from Q.