

Math 337 —Exam 2—f2018

1) (15 points) Let $A = [(3, 6, 1, 0, 7)^T (2, 4, 0, 1, 10)^T (1, 2, 1, -1, -3)^T (0, 0, 0, 3, 12)^T]$ have a reduced echelon form

$$R = [(1, 0, 0, 0, 0)^T (0, 1, 0, 0, 0)^T (1, -1, 0, 0, 0)^T (0, 0, 1, 0, 0)^T].$$

Find bases and dimensions of $\text{Nul}(A)$, $\text{Col}(A)$ and $\text{Row}(A)$.

2) (15 points) Let A be an m by n matrix.

a) Show that $\dim \text{Nul}(A) + \dim \text{Row}(A) = n$.

b) Compute $\dim \text{Nul}(A^T) + \dim \text{Row}(A^T)$.

c) Is there a 5×9 matrix whose null space has dimension 3? Explain.

3) (20 points) Let $A = [(\mathbf{a}, \mathbf{b}, \mathbf{c})^T]$ be a 3×3 matrix with rows $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and let $\det(A) = 2$.

a) Find elementary row operations that transform A to $\mathbf{B} = [(\mathbf{c} + 3\mathbf{b}, 2\mathbf{b}, \mathbf{a})^T]$.

b) Compute $\det(\mathbf{B})$.

c) Compute $\det(2\mathbf{B}A^2(\mathbf{B}^T)^{-1})$.

4) (15 points) Use Cramer's rule to find the solution x_1 of the system

$$2x_1 - 2x_2 + x_3 = 1$$

$$x_1 + x_2 + x_3 = 1$$

$$-x_1 - 2x_2 + x_3 = 1.$$

5) (20 points) a) Find the eigenvalues of $A = [(1, 0, 0)^T (-2, 5, -2)^T (-2, 4, -1)^T]$

b) Find the eigenvectors corresponding to the smallest eigenvalue of A .

c) Find a basis of the eigenspace corresponding to the smallest eigenvalue of A .

6) (15 points) Is H a vector subspace R^3 if

a) $H = \{(x, 2y, x + y)^T \mid x, y \text{ are real}\}$.

b) $H = \{(x, 2y, x + y + 1)^T \mid x, y \text{ are real}\}$.

c) $H = \{(x, y, z)^T \mid y = x^2 + z^2\}$.

Explain your answers.