

**PRACTICE PROBLEMS IN ALGEBRA, TRIGONOMETRY,  
AND ANALYTIC GEOMETRY**

The accompanying problems from the subjects covered on the Mathematics Placement Examination can be used by students to identify subject areas that need attention in preparation for the examination. The examination covers the subjects of Algebra, Trigonometry, and Analytic Geometry which students traditionally learn in high school. It is not expected that all incoming students can work all of the following exercises. The placement examination is used to determine what preparatory courses, if any, are needed to be successful at NJIT. THE USE OF CALCULATORS IS NOT PERMITTED ON THE PLACEMENT EXAM.

1. Evaluate  $\sqrt{25 + 144}$ .
2. Expand  $(x^{\frac{1}{2}} + y^{\frac{1}{2}})^2$ .
3. Combine into a single fraction and simplify:  $\frac{3}{x-1} - \frac{3}{x+1}$ .
4. Combine into a single fraction and simplify:  $\frac{2}{x+3} + \frac{2x+1}{(x+3)^2x}$ .
5. Solve for x:  $x^3 - 6x^2 - 27x = 0$
6. Write and simplify as a fraction without negative exponents:  $\frac{x^2y^{-1}3^{-2}}{x^{-1}y^23^2}$ .
7. Simplify:  $\frac{\frac{2}{x+2} - \frac{1}{x+2}}{\frac{3}{x}}$ .
8. Solve for x:  $-2x^2 - 8x + 1 = 0$ .

In Problems 9,10, and 11, solve for x and check your answers in the original equation:

9.  $x + \sqrt{12x + 25} = -3$
10.  $x + 2 - \sqrt{x} = 4$
11.  $x - 3 = -\sqrt{25 - 12x}$
12. Solve for y:  $\frac{1}{y} = \frac{2}{x} - \frac{3}{5}$ .
13. Combine into a single fraction  $\frac{3}{a} + \frac{3}{b} - \frac{6}{c}$

In Problems 14,15, and 16, *Log* will refer to  $Log_{10}$ , that is Log to the base 10.

14. Solve for x:  $(\text{Log } x) = 2$
15. Solve for x:  $(\text{Log } x)^2 + \text{Log}(x^3) + 2 = 0$
16. Evaluate numerically:  $\text{Log } 5 + \text{Log } 2$
17. Express the solutions for x as Logarithms:  $10^{2x} - 5(10^x) + 6 = 0$ .
- 18a). Simplify:  $(x^3)^2$ .
- 18b). Write as a power of x:  $\sqrt{(x^3)x^2}$ .

In problems 19, 20, and 21, write the given expression in the form:  $a(x + b)^2 + c$ , giving the values of a, b and c.

19.  $x^2 - 4x + 5$

20.  $3x^2 + 5x - 2$

21.  $4x - x^2 + 5$

In problems 22 and 23, describe the set of all x which satisfy the given inequality:

22.  $|2x + 5| < 3$

23.  $|-3x + 1| > 5$

24. Write the expression in terms of  $\text{Log } x$  and  $\text{Log}(x-1)$ :  $\text{Log}\left(\frac{x^2}{\sqrt{x-1}}\right)$ .

25. Solve for x:  $\frac{1}{x} - \frac{1}{2(9-x)} = 0$

26. Solve for x:  $\frac{1}{x} - \frac{x}{4-x^2} = 0$

27. Solve for x:  $-\frac{2}{x^2} + \frac{1}{2(x-3)^2} = 0$

28. Solve for x:  $\text{Log}_{10}(x - 2) = 1$

29. Solve for x:  $\frac{2x(x^2-x)^{-\frac{1}{3}}(2x-1)}{3} + (x^2 - x)^{\frac{2}{3}} = 0$ ;  $x \neq 0$

30. Factor and simplify:  $[x^2 + 3x - 10]^{\frac{3}{2}}$ .

31. Evaluate and simplify the given expression when  $x = 9$ :  $x^{\frac{1}{2}} - x^{\frac{3}{2}}$ .

32. Simplify:  $\frac{x^{-\frac{3}{4}}x^{\frac{3}{10}}}{x^{\frac{1}{12}}}$

33. Simplify:  $\frac{x^{\frac{3}{16}}x^{\frac{3}{4}}x^{-\frac{1}{6}}}{x^{\frac{1}{8}}}$

For problems 34-37, sketch the graph of the function and find the coordinates of the intercepts (if any) with the x and y axes:

34.  $y = (x + 2)^2 + 1$

35.  $y = x^2 - 4x$

36.  $x = 4y^2 - 3y$

37.  $x^2 = 9 - y^2$

38. Evaluate  $\cos(2\text{Arctan}(\frac{3}{4}))$ .

39. Use the quadratic formula to find the roots of  $5x^2 - 13x + 6 = 0$  and then write the expression  $5x^2 - 13x + 6$  in factored form.

40. Given the fact that one root of the given polynomial is  $x = 2$ , Completely factor it.

$$2x^3 - \frac{11}{3}x^2 - x + \frac{2}{3}$$

41. Find the roots of the equation:  $2x^2 + \frac{1}{3}x - \frac{1}{3} = 0$ .

42. Divide  $2x^3 - \frac{11}{3}x^2 - x + \frac{2}{3}$  by  $(x - 2)$ .

43. Find the equation of the line (in  $y = mx + b$  form) through  $(-4, 1)$  with slope  $= -\frac{1}{5}$

44. Find the equation of the line (in  $y = mx + b$  form) with x-intercept = 3 and y-intercept = 5.
45. Solve the system of equations:  $y = -\frac{1}{5}x + \frac{1}{5}$   
 $x = y^2 - 6y + 1$
46. Find the equation of the line (in  $y = mx + b$  form) through the point  $(-2, 5)$  with an angle of inclination of 45 degrees.
47. Find the equation of the line (in  $y = mx + b$  form) through the point  $(-1, 3)$  with an angle of inclination of 120 degrees.
48. Find the equation of the line (in  $y = mx + b$  form) through the point  $(1, -\frac{8}{3})$  and the point  $(-2, \frac{1}{3})$ .
49. Use polynomial division to simplify:  $\frac{\frac{x^3}{3} - x^2 - 2x + \frac{8}{3}}{x^2 + x - 2}$
50. Expand and simplify:  $(x\sqrt{5} + y\sqrt{2})(x\sqrt{5} - y\sqrt{2})$ .
51. Expand  $(\sqrt{2}x - 5)^2$ .
52. Find and simplify the y-coordinate of the point on the curve  $y = x^2 - 4x + 1$  with x-coordinate =  $(\frac{\sqrt{3}+4}{2})$ .
53. Find the x-intercepts of the curve:  $y = x^3 - 16x$ .
54. Find the x-intercepts of the curve:  $y = x^3 - x^2$ .
55. Solve the system of equations:  $y = -x^3 + \frac{3}{4}x^2 + \frac{3}{2}x + \frac{1}{2}$   
 $y = -x^2 + x + \frac{1}{2}$
56. Solve the system of equations:  
 $9a+3b+c=2$   
 $a+b+c=4$   
 $6a+b=0$
57. Find the point of intersection of the lines represented by the equations  $y = 2x - 1$  and  $y = \frac{1}{3}x + 1$ .
58. If the slope of a line through  $(-2, 1)$  is 3, find the equation of the line through  $(-2, 1)$  which is *perpendicular* to this line.
59. Solve for x:  $x^4 - 6x^2 + 8 = 0$ .
60. Find all solution sets to the system of equations:  
 $x^2 + 3y^2 = 13$   
 $2xy = -4$
61. Solve for x:  $(x+3)^3(9x^2 + 6x + 33) - (3x^3 + 3x^2 + 33x + 25)(3)(x+3)^2 = 0$
62. Solve for x:  $(x+3)^4(2)(x-1) - (x-1)^2(4)(x+3)^3 = 0$
63. Evaluate and simplify  $\frac{x^2-2x}{x+1}$  for  $x = -1 + \sqrt{3}$
64. If  $2x^2 + 3x - 2$  is divided by  $(3x-2)$ , find the quotient and remainder.
65. If  $\frac{2x+1}{x^2-5x+6} = \frac{A}{x-3} + \frac{B}{x-2}$ , find A and B.
66. If  $\frac{2x-1}{x(x-2)^2} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$ , find A, B, and C.

67. If  $\frac{10x-20}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$ , find  $A$ ,  $B$ , and  $C$ .
68. Combine into a single fraction and simplify:  $\frac{2}{x-4} - \frac{3}{(x-4)^2} + \frac{2}{x-1}$ .
69. Factor all denominators, find the common denominator, combine into a single fraction and simplify leaving the denominator in factored form:  $\frac{x-1}{x^3-4x} - \frac{1}{x^2-4x+4}$
70. Change 135 degrees to radians.
71. Change 5 radians to degrees (leave the answer in terms of  $\pi$ ).
72. If an arc on a circle of radius 3" subtends a central angle of 30 degrees, find the length of the arc in terms of  $\pi$ .
73. What is the value of  $\sin(240^\circ)$ ?
74. What is the value of  $\cos(870^\circ)$ ?
75. The angle of elevation to the top of a building from a point on the ground 40 feet from the foot of the building is 60 degrees. How tall is the building?
76. Expand the left side of the given equation using the identity for  $\sin(a - b)$  in order to find a value for  $x$  in the second quadrant that satisfies:
- $$\sin\left(x - \frac{\pi}{3}\right) = \frac{1}{2}\sin x + \frac{\sqrt{3}}{2\sqrt{2}}$$
77. Use the identity  $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$  to evaluate  $\sin\left(\frac{-5\pi}{12}\right)$ .
78. Use an appropriate identity for  $\cos 2x$  to write the polar equation  $r = \cos 2\theta$  in Cartesian Coordinates.
79. Put the equation of the line  $x = 3$  into polar coordinates.
80. Sketch the graph of  $y = \tan 2x$ . What is the period of this function?
81. Expand and simplify:  $(x + 1 - \sqrt{2})(x - 1 + \sqrt{2})$
82. In triangle  $ABC$ ,  $a = 5$ ,  $\sin A = .3$  and  $\sin B = .2$ . What is the length of side  $b$ ?
83. Solve for  $x$ :  $\sqrt{x^2 - 9} + 3 = x$
84. Express as a single term in simplest form:  $\sqrt{24} - 6\sqrt{\frac{2}{3}}$
85. Simplify:  $\frac{1+\frac{1}{x}}{\frac{1}{1-x}}$ .
86. Solve for  $x$  in degrees:  $\sin x = \cos(48^\circ)$ .
87. Find the value of  $\sin(\text{Arctan}(\frac{3}{4}))$ .
88. Use the appropriate trigonometric identity to write the expression  $\frac{\cos^2 x}{\sin x} + \sin x$ , as a single term.
89. If  $x = 8$ , what is the value of  $3x^0 - 2x^{-\frac{1}{3}}$ ?
90. Use the appropriate trigonometric identity to expand and simplify  $\sin(180^\circ + x)$ .
91. If  $\sin x = \frac{3}{5}$ , find  $\sin(2x)$ .

92. Solve the system of equations:

$$x^2 + y^2 - 4y - 5 = 0$$

$$x - y - 1 = 0$$

93. Express the value of  $\text{Arcsin}(1)$  in terms of  $\pi$ .

94. If  $x$  is a positive acute angle and  $\cos x = \frac{\sqrt{21}}{5}$ , find the value of  $\sin x$ .

95. In triangle  $ABC$   $b = 6$ ,  $c = 10$  and the measure of angle  $A$  is 30 degrees. Find the area of triangle  $ABC$ .

96. Prove the identity:  $\frac{2\sin^2 x}{\sin 2x} + \frac{1}{\tan x} = (\sec x)(\csc x)$ .

97. Solve the following system of equations:

$$2x + y + z = 2$$

$$4x - 2y - 3z = -2$$

$$8x + 3y + 2z = 5$$

98. If  $x$  and  $y$  are positive acute angles and if  $\sin x = \frac{4}{5}$  and  $\cos y = \frac{8}{17}$ , find the value of  $\cos(x + y)$ .

99. Find the smallest positive value of  $x$  which satisfies the equation  $2\cos^2 x + \cos x - 1 = 0$ .

100. Simplify:  $\frac{2(1+\cos x)}{\sin^2 x + \cos x + \cos^2 x}$ .

101. What is the numerical value of  $\sin \frac{\pi}{6} + \cos \frac{\pi}{2}$ ?

102. Express  $3\cos^2 x + \cos x - 2$  in factored form as a product of two binomials.

103. Find the value of  $\tan(\text{Arcsin} \frac{5}{13})$ .

104. Simplify:  $\frac{\sqrt{50}}{3\sqrt{8}}$ .

105. The length of an arc of a circle is 12". If this arc subtends a central angle of  $1\frac{1}{2}$  radians, find the length of the radius of the circle.

106. The sides of a triangle are 2, 3, and 4. Find the cosine of the largest angle in the triangle.

107. In triangle  $ABC$  the measure of angle  $A$  is 40 degrees greater than that of angle  $C$ , and the measure of angle  $B$  is twice that of angle  $C$ . Find the number of degrees in angle  $C$ .

108. Find the value of  $x$  if  $9^x = 27^2$

109. Express  $\frac{5}{3-\sqrt{7}}$  as an equivalent fraction with a rational denominator.

110. If  $x$  is a positive acute angle whose cosine is  $\frac{1}{2}$  and  $y$  is a positive acute angle whose sine is  $\frac{1}{2}$ . What is the value of  $\sin(x+y)$ ?

111. If  $x$  and  $y$  are angles (as in problem 110), what is the value  $\cos(x - y)$ ?

- 112a). If  $\sin(-x)$  is  $\frac{1}{3}$ , what is the value of  $\sin(x)$  ?
- 112b). If  $\cos(-x)$  is  $\frac{1}{3}$ , what is the value of  $\cos(x)$ ?
113. Simplify:  $\frac{\tan x}{\sec x}$ .
114. The surface area of a sphere varies directly as the square of its radius. If the area is  $36\pi$  when the radius is 3, what is the area when the radius is 6 ?
115. The perimeter of a rectangle is 20". If a diagonal of the rectangle is  $2\sqrt{17}$  ", find the length and width of the rectangle.
116. Find all values of  $x$  from 0 to  $2\pi$  which satisfy the equation  $\cos 2x + \cos x = 0$ .
117. In right triangle ABC the measure of angle C equals 90 degrees and the measure of angle B equals 45 degrees. If the length of side b equals  $\sqrt{3}$ , find the area of the triangle.
118. If the lengths of two sides of a triangle are 7 and 10 and the cosine of the included angle is  $-\frac{1}{7}$ , what is the length of the third side?
119. Determine the solution set for  $2\sin^2 x = 1 + \sin x$  for  $0 \leq x \leq 2\pi$ .
120. What is the period of the function  $y = \cos \frac{1}{2}x$  ?
121. What is the value of  $2\sin 30 - \tan^2 60$  ?
122. What is the value of  $\cos \frac{1}{2}\pi + \sin \frac{\pi}{4}$  ?
123. In a circle with its center at the origin, a central angle of 1 radian subtends an arc of 5 units. What is the equation of the circle?
124. Find the distance between the points (1,-2) and ( 6, 10).
125. Find the coordinates of the midpoint of the line joining (1,-2) and ( 6, 10).
126. Find the center and vertices of the ellipse whose equation is  $9x^2 - 18x + 4y^2 + 16y = 11$ .
127. Find the equations of the asymptotes for the hyperbola with equation:  $\frac{x^2}{4} - y^2 = 1$ .
128. Find the equation of the line which is the perpendicular bisector of the line joining (1,-2) and ( 6, 10).
129. If two points on a circle are (3, 2) and (6, -1), and an equation of a line that goes through the center of this circle is  $y = 2x - 7$ , find an equation for the circle.
130. Find the coordinates of the points at which the parabola  $y = x^2$  intersects the circle with center at the origin and radius equal to 1.
131. An ellipse which is centered at the origin has vertices (4, 0), (0, -2), (-4, 0), (0, 2). Find its equation.
132. A hyperbola which is centered at the origin intersects the y axis at ( 0, 2) and (0, -2). A point on the hyperbola has coordinates  $(\frac{1}{2}\sqrt{5}, 3)$ . Find an equation for the hyperbola.

133. The hypotenuse of a right triangle 25, and one of its sides is 24. Find the length of the other side.
134. The vertices of a triangle are  $A(1, -2)$ ,  $B(4, 6)$ ,  $C(7, -2)$ . Find its area.
135. Solve for  $x$ :  $\log_2(x + 1) = 3$
136. Triangle ABC is a right triangle with angle B=90 degrees. Angle A is 30 degrees. A line is drawn from a point D on line AB to C such that angle CDB is 60 degrees. The length of segment AD is 20. Find the length of side BC.
137. Point A is outside a circle of radius  $r$ . A line is drawn from A to the center of the circle and another line through A tangent to the circle. The distance between A and the circle along the line through A to the center is 4 units. The distance between A and the point of tangency is 12. Find the radius of the circle.
138. Find the center and radius of the circle with equation:  $x^2 - 6x + y^2 + 2y = -1$
139. If the equations of the top and bottom halves of a parabola are  $y = 2 + \sqrt{x + 4}$  and  $y = 2 - \sqrt{x + 4}$  respectively, find a single equation for the entire parabola.
140. Solve the system of equations:  $x \sin y = 3$   
 $x \cos y = \sqrt{3}$
141. Sketch a triangle ABC with vertices  $A(0, -1)$ ,  $B(0, 6)$  and  $C(4, 3)$ . Find the area of this triangle.
142. If  $A$  and  $B$  are angles such that  $\tan A = \frac{1}{3}$  and  $\tan B = \frac{1}{4}$ , find and simplify a numerical value for  $\tan(A + B)$ .
143. Solve for  $x$ :  $2^{x-1} = 2^x - 8$ .
144. A circle with center at  $(2, 3)$  goes through a point with coordinates  $(4, 1)$ . Find an equation for the circle.
145. Find an equation for the axis of symmetry and the coordinates of the vertex point for the parabola  $y = 2x^2 - 8x + 1$ .
146. The interior angles of a triangle are represented by  $x$ ,  $2x + 10$ , and  $\frac{1}{2}x + 30$ . Find the angles of the triangle.
147. If a side of an equilateral triangle has a length of two units, find the area of this triangle.
148. If the equal sides of an isosceles are each 4 units in length and the angles opposite these sides are 30 degrees, find the length of the third side of the triangle.
149. Find an equation for the line which goes through the point  $(1, -3)$  and is parallel to the line  $3y + 4x = 1$ .
150. For  $0 < x < 1$ , find the value of  $y$  if  $y = \text{Arcsin} x + \text{Arccos} x$ .