

MATH 333A: Probability & Statistics. **Examination #2** (Spring 2008)

March 26, 2008 NJIT

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| Name: | Student ID: | Section # |
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Instructors: R. Dios, A. Jain, and V. Venkateswaran

→ **Must show all steps for each problem to receive full credit.**

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I pledge my honor that I have abided by the Honor System. _____

1. A commuter must pass through 3 traffic lights on her way to work. For each traffic light, the probability that it is green when she arrives at the intersection is 0.7. You may assume that the traffic lights and the days are independent. (5 pts each)
 - (a) Find the probability that on a random commuting day, all 3 lights are green.
 - (b) For the next 5 commuting days, let X = number of days that all 3 lights are green. What is the probability distribution of X ?
 - (c) Find $P(X = 2)$
 - (d) Find the mean and variance of X .

2. A film-coating process produces films whose thicknesses are normally distributed with a mean of 100 microns and a standard deviation of 10 microns. For a certain application, the minimum acceptable thickness is 90 microns. (5 pts each)
 - (a) What proportion of films will be too thin?
 - (b) Find the middle symmetric range that contains 50% of film thickness.
 - (c) If the mean of the process could be changed, to what value should the mean be set so that only 1% of the films would be too thin?
 - (d) If the mean of the process could not be changed, to what value should the standard deviation be set so that only 1% of the films would be too thin?

3. The number of cracks on Interstate-80 that are large enough to require repair follows a Poisson distribution, with the mean of 2 cracks per mile. (5 pts each)
 - (a) What is the probability that at least one crack would require repair in $\frac{1}{2}$ mile of the highway?
 - (b) What is the probability that there are four cracks that would require repair in 3 miles of the highway?
 - (c) Let L = length of the highway in miles between two successive cracks. What is the probability density function of L ?
 - (d) Find the $P(1 < L < 2)$.

4. The probability density function (pdf) of a continuous random variable X is:

$$f(x) = k/x^3, \text{ for } x > 1$$

and $f(x) = 0$, otherwise.

- (a) Find the value of k . (5 pts)
 - (b) Find the cumulative distribution function (cdf) of X . (5 pts)
 - (c) Find $P(2 < X < 3)$. (5 pts)
 - (d) Find the mean and the median of X . (5 pts)
5. The lifetime of transistors in an electronic equipment is exponentially distributed with mean of 5 months. (5 pts each)
- (a) Find the probability that a randomly chosen transistor will last longer than 6 months.
 - (b) If a transistor has been operating for 5 months, what is the probability that it will last an additional 4 months?
 - (c) Find the median lifetime of these transistors?
 - (d) If an equipment contains 10 transistors that operate independently, what is the probability that at least 3 of these transistors will last longer than 6 months?

END