

MATH 333A: Probability & Statistics. **Examination #2** (Fall 2008)

Score

November 12, 2008 NJIT

Name:	Student ID:	Section :
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Instructors: R. Dios, A. Jain, and J. Porus

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→ **Must show all steps for each problem to receive full credit.**

I pledge my honor that I have abided by the Honor System. \_\_\_\_\_

(Signature)

- Based on historical records, Kansas can have tornadoes at any time during a year. Suppose that the number of tornadoes ( $X$ ) observed in Kansas during a year follows a Poisson distribution with a mean of 5 tornadoes/year. (5 pts each)
  - Find the probability that during a year there are at least 3 tornadoes.
  - Find the standard deviation of the number of tornadoes in a year.
  - Let  $Y$  denote the number of tornadoes in 3 years. Find the mean of  $Y$ .
  - Find the probability that there are no tornadoes during a 6-month period.
- The diameter of a component produced in a factory follows a normal distribution with mean  $\mu = 1$  inch and standard deviation  $\sigma = 0.1$  inches. A component is considered good if its diameter is between 0.65 and 1.15 inches, otherwise it is defective.
  - What percentage of the components produced would be defective? (5 pts)
  - If you cannot change the standard deviation  $\sigma$  but you can change the mean  $\mu$  from the current value of 1 inch to any value, what should the new value of  $\mu$  be so that the percentage of defective components is reduced to the lowest possible value? (10 pts)
  - After  $\mu$  is changed to the value found in Part (b), what percentage of components be defective? (5 pts)
- Let  $X$  be the difference between the scheduled flight time and the actual flight time (in minutes) from Newark to Miami on Acme Airlines. Suppose  $X$  follows the following probability density function (pdf):
$$f(x) = k(36 - x^2) \text{ for } -6 < x < 6$$
and  $f(x) = 0$ , otherwise.
  - What should the value of  $k$  be for  $f(x)$  to be a legitimate pdf? (5 pts)
  - Find  $F(x)$ , the cumulative distribution function of  $X$ . (5 pts)
  - Find the mean and median of  $X$ . (5 pts)
  - What is the upper 25% point of the distribution of  $X$  (i.e., 25% of the  $X$  values exceed this upper 25% point)? (5 pts)

4. The yield of corn in a one-acre plot follows a normal distribution with a mean  $\mu = 6.75$  tons and a standard deviation  $\sigma = 0.25$  tons. (5 pts each)
- What is the probability that a randomly chosen one-acre plot will have a yield of over 7 tons?
  - What is the probability that a randomly chosen one-acre plot will have a yield of less than 6.5 tons?
  - If a farmer grows corn in 10 one-acre plots, what is the probability that each of these 10 plots will have a yield of over 7 tons?
  - What should be the mean  $\mu$  of the normal distribution of the yield of corn in a one-acre plot, if the probability is 5% that a randomly chosen one-acre plot will have a yield less than 6.5 tons?
5. The time between failures of a machine ( $X$ ) follows an exponential distribution with a mean of 5 days. (4 pts each)
- What is the probability density function (pdf) of  $X$ ?
  - Find the standard deviation of  $X$ .
  - Find the median of  $X$ .
  - After a machine is repaired, what is the probability that it lasts at least a week before it fails again?
  - If a machine has not failed for the last 6 days, what is the probability that it will last at least 2 more days before it fails again?

**END**