

MATH 333A: Probability & Statistics. **Examination #2** (Fall 2008)

Score

November 12, 2008 NJIT

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| Name: | Student ID: | Section : |
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Instructors: R. Dios, A. Jain, and J. Porus

→ **Must show all steps for each problem to receive full credit.**

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I pledge my honor that I have abided by the Honor System. _____
(Signature)

- Based on historical records, Kansas can have tornadoes at any time during a year. Suppose that the number of tornadoes (X) observed in Kansas during a year follows a Poisson distribution with a mean of 5 tornadoes/year. (5 pts each)
 - Find the probability that during a year there are at least 3 tornadoes.
 - Find the standard deviation of the number of tornadoes in a year.
 - Let Y denote the number of tornadoes in 3 years. Find the mean of Y .
 - Find the probability that there are no tornadoes during a 6-month period.
- The diameter of a component produced in a factory follows a normal distribution with mean $\mu = 1$ inch and standard deviation $\sigma = 0.1$ inches. A component is considered good if its diameter is between 0.65 and 1.15 inches, otherwise it is defective.
 - What percentage of the components produced would be defective? (5 pts)
 - If you cannot change the standard deviation σ but you can change the mean μ from the current value of 1 inch to any value, what should the new value of μ be so that the percentage of defective components is reduced to the lowest possible value? (10 pts)
 - After μ is changed to the value found in Part (b), what percentage of components be defective? (5 pts)
- Let X be the difference between the scheduled flight time and the actual flight time (in minutes) from Newark to Miami on Acme Airlines. Suppose X follows the following probability density function (pdf):
$$f(x) = k(36 - x^2) \text{ for } -6 < x < 6$$
and $f(x) = 0$, otherwise.
 - What should the value of k be for $f(x)$ to be a legitimate pdf? (5 pts)
 - Find $F(x)$, the cumulative distribution function of X . (5 pts)
 - Find the mean and median of X . (5 pts)
 - What is the upper 25% point of the distribution of X (i.e., 25% of the X values exceed this upper 25% point)? (5 pts)

4. The yield of corn in a one-acre plot follows a normal distribution with a mean $\mu = 6.75$ tons and a standard deviation $\sigma = 0.25$ tons. (5 pts each)
- (a) What is the probability that a randomly chosen one-acre plot will have a yield of over 7 tons?
 - (b) What is the probability that a randomly chosen one-acre plot will have a yield of less than 6.5 tons?
 - (c) If a farmer grows corn in 10 one-acre plots, what is the probability that each of these 10 plots will have a yield of over 7 tons?
 - (d) What should be the mean μ of the normal distribution of the yield of corn in a one-acre plot, if the probability is 5% that a randomly chosen one-acre plot will have a yield less than 6.5 tons?
5. The time between failures of a machine (X) follows an exponential distribution with a mean of 5 days. (4 pts each)
- (a) What is the probability density function (pdf) of X ?
 - (b) Find the standard deviation of X .
 - (c) Find the median of X .
 - (d) After a machine is repaired, what is the probability that it lasts at least a week before it fails again?
 - (e) If a machine has not failed for the last 6 days, what is the probability that it will last at least 2 more days before it fails again?

END