

Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. Always simplify when possible. No calculators!

- (10 points) Find a parametric equation of the line of intersection of the planes $x + y = 4$ and $2x - y - z = 2$.
- (10 points) Find the velocity and the position vector of a particle whose acceleration vector is $\mathbf{a}(t) = \cos 2t \mathbf{i} + \sin 2t \mathbf{j} + \mathbf{k}$ and the initial velocity and position are $\mathbf{v}(0) = \mathbf{i}$ and $\mathbf{r}(0) = \mathbf{j}$, respectively.
- (10 points) Use differentials to estimate the maximum error in the calculated volume of a rectangular box, if its dimensions are measured to be 10 cm, 20 cm, and 30 cm, respectively, each within an error of at most 0.1 cm. Write the answer as the calculated volume \pm the estimated error.

- (10 points) Find and classify all the critical points of the function

$$f(x, y) = x^2 + 3xy - y^2 + 16x - 2y + 5.$$

- (10 points) Use Lagrange Multipliers to find the minimum value of the function $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $x + y - z = 5$.
- (a) (10 points) Use the Fundamental Theorem of line integrals to compute

$$\int_{(1,2,3)}^{(3,2,1)} yz \, dx + xz \, dy + xy \, dz.$$

- (b) (10 points) Verify the answer in part (a) by explicitly computing the integral over a straight line segment connecting $(1, 2, 3)$ with $(3, 2, 1)$.
- (10 points) Evaluate the surface integral

$$\iint_S y \, dS,$$

where S is the surface $z = x + y^2$, $0 \leq x \leq 1$, $0 \leq y \leq 2$.

- (10 points) Use Stoke's Theorem to evaluate

$$\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S},$$

where $\mathbf{F}(x, y, z) = x^2 e^{yz} \mathbf{i} + y^2 e^{zx} \mathbf{j} + z^2 e^{xy} \mathbf{k}$, and S is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$, oriented upward.

- (10 points) Use the Divergence Theorem to calculate the flux of the vector field \mathbf{F} across the surface S , that is, $\iint_S \mathbf{F} \cdot \mathbf{n} \, dS$, where $\mathbf{F}(x, y, z) = x^2 y \mathbf{i} + xy^2 \mathbf{j} + 2xyz \mathbf{k}$, S is the surface of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + 2y + z = 2$, and \mathbf{n} is the outward normal to S .