Math 213 **Exam 3** November 29, 2006

Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. Always simplify when possible. No calculators!

1. (16 points) Compute:
$$\iint_R (2x - 3)\, dxdy,$$
where R is the region enclosed by the curves $y = x + 4$ and $y = x^2 - 2x$.

2. (16 points) Compute the volume of a solid enclosed by the following surfaces:
$$z = 4\sqrt{x^2 + y^2}, \qquad z = 5 - x^2 - y^2, \qquad x = 0.$$
(*Hint:* use cylindrical coordinates).

3. (16 points) Let $\mathbf{F} = y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$. Use the Fundamental Theorem of line integrals to compute
$$\int_{(0,1,-2)}^{(-1,-1,0)} \mathbf{F} \cdot d\mathbf{r}.$$

4. (16 points) Use Green's Theorem to compute
$$\oint_C -\tfrac{1}{4}ydx + \tfrac{3}{4}xdy,$$
where C is the path consisting of straight line segments connecting $(0, 0)$, $(2, -1)$, $(0, 1)$, back to $(0, 0)$. Interpret this integral geometrically.

5. (18 points) Compute
$$\iint_D (3y - 2x)\, dxdy, \qquad D = \{(x, y) |\ \ 0 \le x + 2y \le 7,\ \ -7 \le y - 3x \le 7\},$$
using the transformation $x = u - 2v$, $y = 3u + v$.

6. (18 points) Use spherical coordinates to compute:
$$\iiint_E e^z\, dxdydz, \qquad E = \{(x, y, z) |\ z \ge 0,\ x^2 + y^2 + z^2 \le 9\}.$$