Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. Always simplify when possible. No calculators!

1. (18 points)

   (a) Use the Chain Rule to find the partial derivatives $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$, where
   $$w = 3xy + y^3 - 1, \qquad x = 2u + v^2, \qquad y = u^3 - 3v,$$
   when $u = 1$ and $v = 2$.

   (b) Integrate:
   $$\int_{-1}^{2} \int_{3}^{6} (2x^2y - 3x)\, dy dx.$$

2. (10 points) Find the velocity and position of a particle that has the given acceleration and the given initial velocity and position:
$$\mathbf{a}(t) = 2t\,\mathbf{i} - 10\,\mathbf{k}, \qquad \mathbf{v}(0) = 3\,\mathbf{j}, \qquad \mathbf{r}(0) = \mathbf{0}.$$

3. (14 points) Find an equation of the tangent plane to the graph of the function
$$f(x, y) = (x + 2y)\cos(3xy)$$
at the point $(0, 1, 2)$.

4. (14 points) Let $g(x, y, z) = e^z - xy^2 + z\sin(\pi x)$.

   (a) Find the directional derivative of g in the direction of the vector $\mathbf{v} = \langle 1, 1, 1\rangle$ at the point $P(1, -2, 0)$.

   (b) In which direction does g have the maximum rate of change at that point? What is its maximum rate of change there?

5. (14 points) The total resistance R of two resistors with resistances R_1 and R_2 connected in parallel is given by the following well-known formula:
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$
Suppose that R_1 and R_2 are measured to be 300 and 600 ohms, respectively, with an error of ± 15 ohms in each measurement. Use differentials to estimate the maximum error in ohms in the calculated value of R.

6. (15 points) Find all the local minima, maxima, and saddle point points of the function
$$f(x, y) = 5x^2 - 4xy + y^2 - 4x + 6y + 10.$$

7. (15 points) Use the method of Lagrange Multipliers to find the minimum and the maximum value of the function $f(x, y, z) = xy + yz$ subject to the constraints $xy = 1$ and $y^2 + z^2 = 1$.