Doctoral Qualifying Exam, Linear Algebra and Numerical Methods.

Fall, 1999.

1. Determine the order of the local discretization error $O(h^n)$, i.e., find the value of n for the following methods:
(a) The method

$$y_{k+1} = y_k + hf(x_k + \frac{h}{2}, y_k + \frac{h}{2} f(x_k, y_k))$$

known as the mid-point rule.

(b) The method

$$y_{k+1} = y_k + \frac{h}{2}(3f_k - f_{k-1})$$

known as the 2-step Adams-Bashforth method.

2. Determine the values of n and h, where the local discretization error is defined to be $O(h^n)$, needed to approximate the integral

$$\int_0^2 \frac{1}{x+4}\, dx$$

to within 10^{-5} using the composite Simpson's rule.

(b) Compute an approximation to this integral using the composite Simpson's rule with the values of n and h you found in part (a).

(c) Is your answer in part (b) within 10^{-5} of the exact answer?

3. The iteration $x_{n+1} = 2 - (1+c)x_n + cx_n^3$ will converge to $\alpha = 1$ for some values of c, provided x_0 is sufficiently close to α. Find the values of c for which this is true. For what values of c will the convergence be quadratic?

4. (a) A middle-aged man was stretched on a rack to lengths $L = 5, 6,$ and 7 under applied forces of $F = 1, 2,$ and 4 tons. Assuming Hooke's law $L = a + bF$, find the man's normal length a using least squares.

(b) Prove that the projection matrix $P = A(A^T A)^{-1} A^T$ has the properties: (i) $P^T = P$, (ii) $P^2 = P$.

5. (a) Let A, B, and M be $n \times n$ non-singular matrices. If $B = M^{-1}AM$, how are $\det(A)$ and $\det(B)$ related?

(b) If B is similar to A and C is similar to B, show that C is similar to A.

6. (a) Show that any positive definite $n \times n$ matrix A can be written as $A = B\,B^T$, where B is a $n \times n$ matrix with orthogonal columns.

(b) Show that any positive definite 2×2 matrix A can be written uniquely as $A = L\,L^T$, where L is a lower-triangular 2×2 matrix with positive entries on the diagonal.