Math 112 – Final Examination

Please complete the following problems. All work must be shown in order to receive full credit. Answers without explanation will receive no credit. The use of books, notes, calculators, or any other external sources of information is not allowed during this examination.

1. (22 pts.) Consider the parametric curve that is given by $x(t) = \cos^2(t), y(t) = \sin^2(t)$ for $0 \leq t \leq \frac{\pi}{4}$.
   a. Find the length of this curve.
   b. Find the area of the surface generated by revolving this curve about the $x$-axis.

2. (22 pts.) Consider the polar curve given by $r = 4 \sin(\theta)$.
   a. Find the slope of the tangent line to this curve for $\theta = \frac{\pi}{3}$.
   b. Find the area enclosed by this curve.

3. (18 pts.) Evaluate the following integrals:
   a. $\int \frac{4x + 1}{x(x + 1)^2} \, dx$
   b. $\int \frac{1}{(4 - x^2)^{\frac{3}{2}}} \, dx$

4. (18 pts.) Evaluate the following integrals:
   a. $\int \frac{3x - 1}{x^3 + x} \, dx$
   b. $\int \frac{\ln(x)}{\sqrt{x}} \, dx$

5. (18 pts.) Evaluate the following improper integrals if they are convergent or show they are divergent:
   a. $\int_0^1 \frac{\cos(\sqrt{x})}{\sqrt{x}} \, dx$
   b. $\int_0^\infty xe^{-x} \, dx$

6. (16 pts.) Find the volume of the solid obtained by revolving the region enclosed by $y = \frac{1}{x^2 + 1}, y = 0, x = 0$ and $x = 1$ about the $x$-axis.

7. (26 pts.) Determine whether the following series are convergent or divergent. If you use a convergence or divergence test, please state which test you are using.
   a. $\sum_{n=1}^{\infty} \frac{n + 1}{\sqrt{n^4 + 4}}$
   b. $\sum_{n=1}^{\infty} \frac{1}{e^n + 1}$
   c. $\sum_{n=1}^{\infty} \frac{2^n + 5^n}{4^n + 5^n}$
8. (16 pts.) Find the radius of convergence and interval of convergence for \( \sum_{n=1}^{\infty} \frac{3^n(x - 1)^n}{n} \).

9. (22 pts.) Consider the function \( f(x) = x \cos(x^2) \).

   a. Find the Maclaurin series (Taylor series about \( a = 0 \)) for \( f(x) \).

   b. Use the series from part a to evaluate the following: \( \lim_{x \to 0} \frac{x \cos(x^2) - x}{3x^5} \).

10. (22 pts.) Consider the function \( f(x) = e^x \).

   a. Find the first four nonzero terms of the Taylor series about \( a = 2 \) for \( f(x) \).

   b. Suppose \( f(x) \) is approximated by the Taylor polynomial from part a. Estimate the error when \( 1 \leq x \leq 3 \).