MATH 337 —–-FINAL EXAM—–-SPRING 2011

Justify all your answers.

1.(15 points) a) Let $A = [v_1 v_2 v_3 v_4]$, where $v_1 = (1,0,1)^T, v_2 = (3,0,3)^T, v_3 = (0,1,1)^T$, $v_4 = (2,4,6)^T$, and $b = (1,6,7)^T$. Find the general solution of Ax=b.

b) Are the columns of Col(A) linearly independent?

c) Is the system $Ax = b$ solvable for each b in R^3 ?

2)(15 points) a) Let $A = [v_1 v_2 v_3 v_4]$, where $v_1 = (2,3)^T, v_2 = (0,4)^T, v_3 = (-3,2)^T$, $v_4 = (1,2)^T$. Find bases of Nul(A), Col(A) and Row(A).

b) What is the rank of A? Is the Nul(A) orthogonal to Col(A) ? Explain.

3) (15 points) Let $u = (2,1,0)^T$, $v = (1,1,1)^T$ and $V = \text{span}\{v\}$.

a) Compute $w = \text{proj}_V u$.

b) Write u as the sum of w and a vector orthogonal to v.

c) Find the distance from u to V.

4) (20 points) Let $A = [v_1 v_2 v_3]$, where $v_1 = (1,1,1)^T, v_2 = (1,1,1)^T, v_3 = (1,1,1)^T$.

a) Find the eigenvalues of A.

b) Find bases of the corresponding eigenspaces.

c) Diagonalize A (i.e, write it as $A = PDP^{-1}$). Do not compute P^{-1}.

d) Using part c), compute det(A). Is A invertible?

5) (20 points) Let $A = [v_1 v_2 v_3]$, where $v_1 = (1,1,0)^T, v_2 = (1,0,1)^T, v_3 = (1,0,0)^T$.

a) Use the Gram-Schmidt method to find an orthogonal basis for V=Col(A).

b) Find the QR factorization of A.

c) Is A invertible? Is the system $Ax = b$ solvable for each b in R^3? Give the formula for its solution(s) when solvable? Justify your answer.

6) (15 points) a) Write down of the matrix A corresponding to the quadratic form $Q(x) = x_1^2 + 2x_1x_2 + 2x_1x_3$.

b) Is Q positive definite, negative definite, or indefinite?

c) Orthogonally diagonalize A and find $2A^5$ and $\det(2A^5)$.