Read each problem carefully. Show all your work for each problem. No Calculators!

1. (a) (8) Determine if the functions \( y_1, y_2 \) are linearly dependent or independent:
   
   (i) \( y_1 = |t - 1|, \ y_2 = 2(t - 1) \), \( (ii) \ y_1 = 3t + 1, \ y_2 = t + 3 \)
   
   (b) (8) Find a function \( g(x) \) which satisfies the conditions: \( W(f, g) = x, \ f(x) = x \).

2. (a) (12) Use the method of undetermined coefficients to find a particular solution of the differential equation
   
   \[ y'' - y' = 2e^t - 1 - t \]
   
   (b) (6) Determine the general solution of the above equation

3. (a) (12) Given that \( y_1 = e^{-x} \) is a solution of the differential equation
   
   \[ xy'' + (x - 1)y' - y = 0, \ x > 0, \]

   use the method of reduction of order to find the second linearly independent solution \( y_2 \).

   (b) (6) Determine the homogeneous ODE whose general solution is
   
   \[ y = c_1e^t + c_2te^t + e^{-t}(c_3cos2t + c_4sin2t) \]

4. (16) Use the method of variation of parameter to find a particular solution of the differential equation
   
   \[ 2y'' + 4y' + 2y = \frac{1}{t}e^{-t}, \ t > 0 \]

5. (16) Determine the form of particular solution of the following ODE, using the method of undetermined coefficients. Do NOT evaluate the constants.
   
   \[ y^{(4)} + 2y^{(3)} + 2y'' = 4e^t - 2e^{-t}cos(t) + te^{-t} \]

6. (16) Solve the initial value problem
   
   \[ y^{(3)} - y'' - y' + y = 0, \ y(0) = 2, \ y'(0) = -1, \ y''(0) = 0 \]