Math 222 EXAM I,  September 26, 2007

Read each problem carefully. Show all your work for each problem. No Calculators!

1. (a) (8) Determine the order of each differential equation and whether it is linear or nonlinear:
   (i) \( t^3 y'' - t(1 + y')y = 2t^2 \ln t \),  
   (ii) \( dy/dt + e^t(t + 1)/y = 0 \)

(b) (8) Solve the IVP: \( y'' + y' - 6y = 0, \ y(0) = 1, \ y'(0) = k \) and find the value of \( k \) for which the solution approaches zero as \( t \to \infty \).

2. (a) (8) Solve the differential equation: \( te^y y' = e^y + 1, \ t > 0 \)

(b) (8) Determine all values of \( r \) for which the differential equation

\[
(t-1)^2 y'' - 2y = 0, \ t > 1,
\]

has a solution of the form \( y = (t-1)^r \).

3. (a) (8) Find constants \( a \) and \( b \) such that all solutions of the differential equation \( y' = ay + b \) converge to \( y = 1/2 \) as \( t \to \infty \)

(b) (8) Write a differential equation whose general solution is \( y = c_1e^{9t} + c_2e^{-2t} \)

4. (a) (8) Determine (without solving the problem) the longest interval in which the following IVP is certain to have a unique solution, according to the Existence and Uniqueness Theorem for linear equations. State reason for your answer.

\[
(t^2 - 9)y' + y \ln(t + 1) = 2 \sec t, \ y(0) = 5
\]

(b) (8) Solve the differential equation: \( dy/dt + t^{-1}y = \cos(t), \ t > 0 \)

5. (a) (10) Solve the IVP: \( y' = e^{2y}, \ y(0) = 0 \)

(b) (8) Determine the interval in which the solution exists.

6. (a) (10) Solve the IVP: \( 2y' - y^2 + 4 = 0, \ y(0) = 0 \)

(b) (8) Determine the behavior of the solution for large \( t \) (i.e., \( \lim_{t\to\infty} y(t) \)).