

**Math 222 EXAM I, September 26, 2007**

Read each problem carefully. Show all your work for each problem. No Calculators!

1. (a) (8) Determine the order of each differential equation and whether it is linear or nonlinear:

(i)  $t^3 y'' - t(1 + y')y = 2t^2 \ln t$ , (ii)  $dy/dt + e^t(t + 1)/y = 0$

- (b) (8) Solve the IVP:  $y'' + y' - 6y = 0$ ,  $y(0) = 1$ ,  $y'(0) = k$  and find the value of  $k$  for which the solution approaches zero as  $t \rightarrow \infty$ .

2. (a) (8) Solve the differential equation:  $te^y y' = e^y + 1$ ,  $t > 0$

- (b) (8) Determine all values of  $r$  for which the differential equation

$$(t - 1)^2 y'' - 2y = 0, \quad t > 1,$$

has a solution of the form  $y = (t - 1)^r$ .

3. (a) (8) Find constants  $a$  and  $b$  such that all solutions of the differential equation  $y' = ay + b$  converge to  $y = 1/2$  as  $t \rightarrow \infty$

- (b) (8) Write a differential equation whose general solution is

$$y = c_1 e^{9t} + c_2 e^{-2t}$$

4. (a) (8) Determine (without solving the problem) the longest interval in which the following IVP is certain to have a unique solution, according to the Existence and Uniqueness Theorem for linear equations. State reason for your answer.

$$(t^2 - 9)y' + y \ln(t + 1) = 2 \sec t, \quad y(0) = 5$$

- (b) (8) Solve the differential equation:  $dy/dt + t^{-1}y = \cos(t)$ ,  $t > 0$

5. (a) (10) Solve the IVP:  $y' = e^{2y}$ ,  $y(0) = 0$

- (b) (8) Determine the interval in which the solution exists.

6. (a) (10) Solve the IVP:  $2y' - y^2 + 4 = 0$ ,  $y(0) = 0$

- (b) (8) Determine the behavior of the solution for large  $t$  (i.e., find  $\lim_{t \rightarrow \infty} y(t)$ ).