Math 222 EXAM II, March 7, 2012

Read each problem carefully. Show all your work for each problem! No Calculators!

1. (16) Find the general solution of each of the differential equations:
   (a) \( y'' + 2y' + 17y = 0 \)  
   (b) \( 4y'' + 4y' + y = 0 \)

2. (16) Verify that \( y_1 = t \) is a solution of the ODE \( t^2y'' + ty' - y = 0, \ t > 0, \)
   and find a second linearly independent solution \( y_2 \) using the method of
   reduction of order.

3. A mass weighing 8 lb stretches a spring 2 ft. Assume there is no damping or external forces
   acting on the system. Suppose the mass is pulled down 1 ft below its equilibrium position, and
   released with an upward velocity of 4 ft/s.
   (a) (8) Determine the position \( y(t) \) of the mass at any time \( t \).
   (b) (8) Find the amplitude, phase angle and period of the motion.

4. (16) Solve the IVP: \( y'' - 2y' + 17y = 0, \ y(\pi/4) = 1, y'(\pi/4) = -1 \).

5. Consider the ODE: \( y'' - 2y' + 2y = te^t + e^{2t}\sin2t \)
   (a) (6) Find the fundamental set of solutions of the corresponding homogeneous equation.
   (b) (12) Write a suitable form for the particular solution \( y_p \) if the method of undetermined
            coefficients is to be used, but do NOT evaluate the coefficients in the form of \( y_p \).

6. Consider the ODE: \( (1-t)y'' + ty' - y = 2(t-1)^2e^{-t}, \ 0 < t < 1 \).
   (a) (12) Assume that \( y_1 = e^t \) and \( y_2 = t \) form a fundamental set of solutions of the
            corresponding homogeneous equation and find the particular solution \( y_p \) by the
            method of variation of parameter.
   (b) (6) Write the general solution of the above ODE.