

Math 222 EXAM I, February 8, 2012

Read each problem carefully. Show all your work for each problem! No Calculators!

1. (a) (10) Without solving the problem, find the largest interval over which the solution of the following initial value problem is certain to exist

$$(t + 2)(t - 4)y'' + e^t y' + (\ln t)y = t^2, y(1) = 1, y'(1) = -3$$

- (b) (6) For each of the differential equations determine its order, and whether the equation is linear or nonlinear:

$$(i) 2x y''' + e^{2x} y' + \sin(x + y) = x^2, \quad (ii) \frac{d}{dt}(t \frac{dy}{dt}) + y \cos t - 1 = 0.$$

2. (16) The population of bacteria in a given culture grows at a rate proportional to the number of bacteria present at any time. After 3 hours it is observed that number of bacteria is 400 and after 10 hours the number has increased to 2000. What was the initial number of bacteria in that culture?

3. (a) (6) Find constants a and b such that all solutions of ODE $y' = ay + b$ converge to $y = 3$.

- (b) (10) Solve the initial value problem: $x \frac{dy}{dx} - y = x, y(1) = 1$.

4. (16) Find α so that the solution of the following IVP approaches zero as $t \rightarrow \infty$

$$y'' - 2y' - 3y = 0, y(0) = \alpha, y'(0) = 1.$$

5. A 120-gallon tank initially contains 90 lb of salt dissolved in 90 gal of water. A solution containing 2 lb/gal of salt flows into the tank at the rate of 4 gal/min, and the well-stirred mixture flows out of the tank at the rate of 3 gal/min.

- (a) (4) Find the time t_1 when the tank begins to overflow.

- (b) (12) Determine the amount of salt in the tank at any time t prior to overflow.

6. (a) (4) Verify that $y_1 = t^{-3}$ is a solution of the differential equation

$$t^2 y'' + 3t y' - 3y = 0.$$

- (b) (12) Find a second solution y_2 if it is known that the Wronskian $W[y_1, y_2] = 4t^3$.

- (c) (4) Are the two solutions linearly independent? Give reason for your answer.