Math 222 Exam 2, March 11, 2015

Read each problem carefully. Show all your work for each problem. No calculators!

1. (a) (6) Evaluate the Wronskian of $\sin 2x$ and $\sin x \cos x$. Are these functions linearly dependent or independent?
   (b) (6) Without solving the problem, find the largest interval over which the solution of the initial value problem $(x-3)y'' + xy' + (\ln|x|)y = 0$, $y(1) = 0$, $y'(1) = 1$ is certain to exist.

2. (a) (10) Find a fundamental pair of solutions of $y'' - 4y' + 4y = 0$, and give the general solution. Also, solve the initial value problem when $y(0) = 1$ and $y'(0) = 1$.
   (b) (10) Solve the initial value problem $y'' + 2y' + 10y = 0$, $y(0) = 2$, $y'(0) = 1$.

3. (16) Show that $y_1 = t$ is a solution of $t^2y'' + t(t-2)y' + (2-t)y = 0$, and find a second solution by reduction of order.

4. (a) (9) Use the method of undetermined coefficients to find the general solution of

$$y'' + 3y' + 2y = 2x^2 + 6x + 10\cos x\,.$$

   (b) (9) Determine a suitable form (or ansatz) if the method of undetermined coefficients is used to find the particular solution y_p of the ODE below. Do NOT attempt to evaluate the coefficients

$$y'' + 5y' + 6y = 2t^2 + te^{-2t} + 4t\cos 3t\,.$$

5. (16) Use the method of variation of parameters to find the general solution of

$$y'' - 2y' + y = \frac{e^t}{1+t^2}\,.$$

6. A force of 2.5 N stretches a spring 0.1 m. A mass of 1 kg is hung from the spring and is attached to a viscous damper that exerts a force of 3 N when the velocity of the mass is 0.5 m/s. The mass is pulled down 0.4 m below its equilibrium position $u = 0$ and then released from rest.
   (i) (5) Formulate the initial value problem for this system.
   (ii) (10) Find the displacement u of the mass at any time t. Sketch the graph of u versus t and describe the motion.
   (iii) (3) Express the solution in the form $u = e^{-\kappa t}R\sin(\omega t + \delta)$.