Math 211 Final Exam Spring 2015

Name: ____________________

Instructions: Show all work and justify all steps of each argument you make. Points may be deducted if either is missing or inadequate. Note that there is one question on the back. You have $2\frac{1}{2}$ hours for this exam.

1. (20 points) Find the equation of the plane described by points $P(2,-1,1)$, $Q(0,1,0)$, and $R(1,1,-2)$, and find the distance from the point $S(0,1,-1)$ to that plane.

2. (20 points) Find the tangent plane and normal line of the surface

$$f(x,y,z) = 4x^2 - 8x + y^2 + 2y - 2xyz - 3 = 0$$

at the point $P_0(1,1,-2)$.

3. (20 points) Find the point on the plane $x+2y+3z = 13$ closest to the point $(1,1,1)$.

4. (20 points) Sketch the region of integration for $\int_0^4 \int_{\sqrt{y}}^2 e^{x^3}\,dx\,dy$ and write an equivalent double integral with the order of integration reversed. Evaluate the integral.

5. (20 points) Use a triple integral in cylindrical coordinates to find the volume bounded by $z = 4 - x^2 - y^2$ and $z = x^2 + y^2 - 4$.

6. (20 points) Find the centroid of the region in the first quadrant bounded by the x-axis, $x = y^2$, and the line $x + y = 2$.

7. (20 points) Evaluate the line integral $\int_C x + \sqrt{y} + z^2\,ds$ over the straight line segment from $(1,2,3)$ to $(0,1,1)$.

8. (30 points) First, show that the vector field $\mathbf{F} = y\sin z\,\mathbf{i} + x\sin z\,\mathbf{j} + xy\cos z\,\mathbf{k}$ satisfies the conditions necessary for it to be conservative. Second, find a potential function for this field. Third, evaluate the line integral $\int_{(1,1,0)}^{(1,1,\pi/2)} \mathbf{F}\cdot d\mathbf{r}$.

9. (a) (15 points) Apply Green's theorem to evaluate $\oint_C \mathbf{F}\cdot\mathbf{n}\,ds$, where $\mathbf{F} = (x+y)\,\mathbf{i} - (x^2+y^2)\,\mathbf{j}$, $\mathbf{n}$ is the outward-pointing normal vector on C, and C is the boundary of a triangle with vertices $(1,0),(0,1),(-1,0)$ oriented in the counterclockwise direction.

   (b) (15 points) Evaluate directly the line integral $\oint_C \mathbf{F}\cdot\mathbf{n}\,ds$ in part (**a**).