

Math 111 – Spring 2014
Examination 3

Please complete the following problems. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, or any other external sources of information is not allowed during this examination.

1.(15 pts.) Find the most general antiderivative for the following:

a. $f(x) = e^{-x} + \sec^2(2x)$ b. $f(x) = \left(1 - \frac{1}{x}\right)^2$

c. $f(x) = \frac{x^e - \sqrt{x}}{x}$

2.(15 pts.) Evaluate the following limits:

a. $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$ b. $\lim_{x \rightarrow 0} \frac{\tan(x)}{\arctan(x)}$

c. $\lim_{x \rightarrow 0^+} (1 + x)^{\frac{1}{x}}$

3.(7 pts.) Use Newton's method to find $\sqrt[3]{4}$ by estimating the zeros of $f(x) = x^3 - 4$. Start with $x_0 = 1$ and find x_2 .

4.(7 pts.) Find the linearization of $f(x) = \sqrt{1+x}$ about $a = 3$.

5.(8 pts.) An open-top rectangular tank with a square base and a volume of 32 ft³ is to be built. What dimensions minimize the amount of material required to build this tank? Show that your result is a minimum.

6.(12 pts.) Find the absolute maximum and absolute minimum values of each function on the given interval.

a. $y = x\sqrt{18 - x^2}$, $0 \leq x \leq 4$ b. $y = \sqrt[3]{x^2 - 1}$, $-3 \leq x \leq 3$

7.(16 pts.) Consider the function $y = 2 + 3x^2 - x^3$.

- a. Find the intervals on which this function is increasing or decreasing.
- b. Find the intervals on which this function is concave up or concave down.
- c. Determine the points at which this function has a local maximum, a local minimum, or a point of inflection.
- d. Sketch a graph of this function making sure to label the points found in part c.

8.(20 pts.) Consider the function $y = \frac{x}{1 - 2x}$.

- a. Find all asymptotes of this function.
- b. Find the intervals on which this function is increasing or decreasing.
- c. Find the intervals on which this function is concave up or concave down.
- d. Determine the points (if any) at which this function has a local maximum, a local minimum, or a point of inflection.
- e. Sketch a graph of this function making sure to label the asymptotes from part **a** and the points found in part **d**.