Math 110 Final Exam
December 18, 2015

Time: 2 hours and 30 minutes
Instructions: Show all work for full credit. No outside materials or calculators allowed.
Extra Space: Use the backs of each sheet for extra space. Clearly label when doing so.

Name: ______________________________

ID #: ______________________________

Instructor/Section: ____________________

"I pledge my honor that I have abided by the Honor System."

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(Signature)

Line	Problem	Score	Running Total
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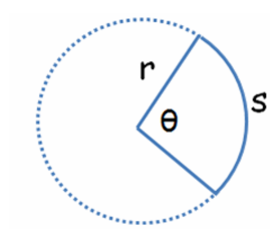
<u>Relevant Formulas for this Exam:</u>

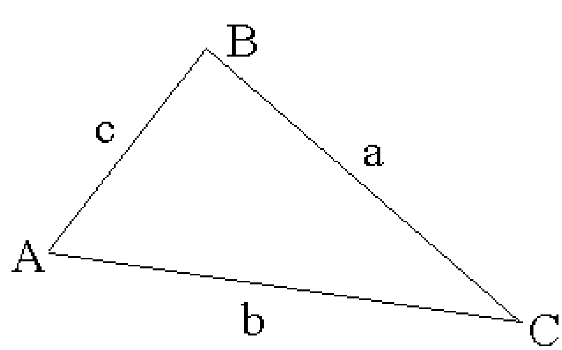
Circular motion and equations relating to a sector of a circle, radius r (as shown to the right).



$A = \frac{1}{2}\theta r^2$ (where A is the area of the sector cut out by θ)
$s = r\theta$ (where s is the arc length as shown)
$v = \omega r$ (where v is velocity and ω is angular velocity)

sin(A+B) = sin(A)cos(B) + sin(B)cos(A)
sin(A–B) = sin(A)cos(B) – sin(B)cos(A)
cos(A+B) = cos(A)cos(B) – sin(A)sin(B)
cos(A–B) = cos(A)cos(B) + sin(A)sin(B)

Given ΔABC as shown to the right:



$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$b^2 = a^2 + c^2 - 2ac\cos(B)$$
$$c^2 = b^2 + a^2 - 2ab\cos(C)$$

Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = (a+b+c)/2$

1. Solve the following equations for all real solutions in the domain **(12 points)**:

a. $\frac{2}{3}x - \frac{1}{2} = \frac{3}{5}x$

b. $\log_2 x + \log_2 (x-2) = 3$

c. $x^3 - 2x^2 = 10x$

e. $-e^{3x} + e^{\ln(4)} = (e^x)^3$

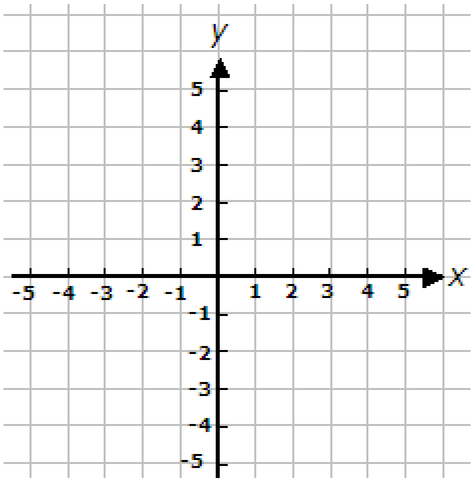
3a. Simplify completely: $\dfrac{\sqrt{4t^2-4}}{2t-2}$ **(3 pts)**

3b. Solve for x and rationalize: $\sqrt{5}=\dfrac{2-x}{x}$ **(3 pts)**

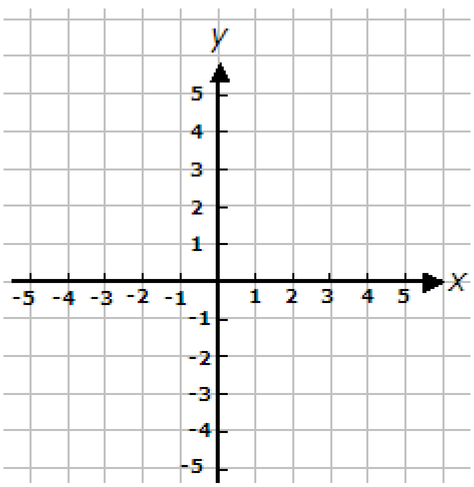
4. Suppose ΔABC has side lengths of a = 5 and b = 8 while $\angle C = 60^\circ$. Use this information to find $\angle A$ and the length of side 'c'. You may leave answers as inverse trig functions if necessary. **(6 points)**

5. Graph the following equations on the coordinate axis given below **(12 points)**.

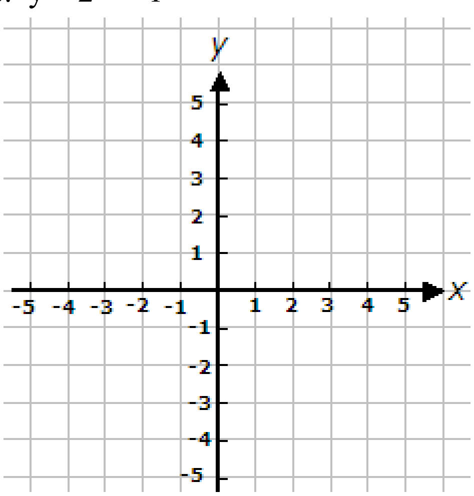
a. $4y^2 - 8y + 9x^2 = 32$



b. $3(y-1) = (x+3)$



c. $y = 2^{-x} - 1$



6. Given the function $f(x) = x^2 - x$, evaluate and simplify the quantity: $\frac{f(2t)}{2} - f(t+1)$

(4 points)

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7. The temperature variation in a certain location in the Gobi Desert is modeled by the following formula where H(t) is the temperature (in degrees Celsius) and t is the time in hours after midnight. **(12 points)**

$$H(t) = 26 + 12\sin\left[\frac{\pi}{12}(t+14)\right]$$

a. What is the period of this temperature cycle?

b. What is the minimum temperature that is reached in the Gobi Desert.

c. At what time does this minimum temperature occur?

d. What is the temperature at 8:00 a.m.

8. Suppose that a pulley with a radius of 4 inches is rotating such that a point on the rim of the pulley is moving at a velocity of 3 feet/second. Find how many revolutions per minute this point is making. **(5 points)**

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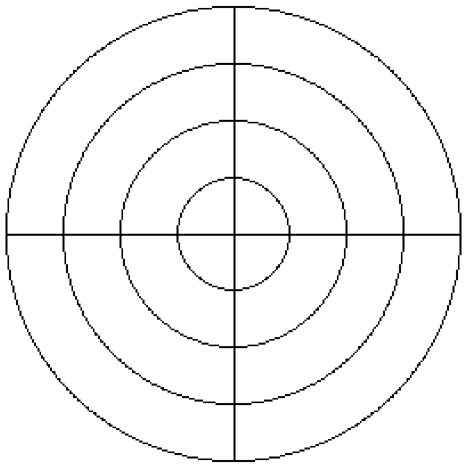
8. Suppose that a given triangle has sides of length 4, k and k (where k is a constant).

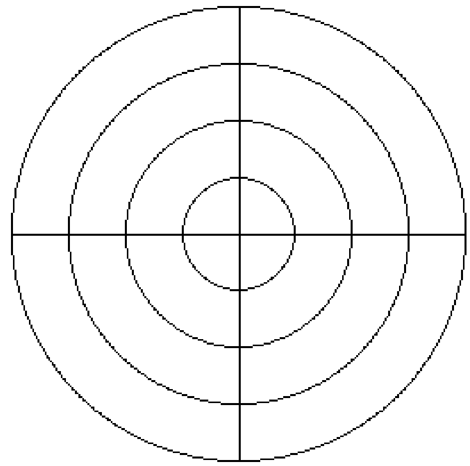
a) Find the area of such a triangle in terms of k. **(4 points)**
b) For what value(s) of k, if any, would this triangle be a right triangle? **(2 pts)**

9. Graph the following graphs on the polar axis given below **(8 points)**:

a. $r = 2 - 2\cos(\theta)$



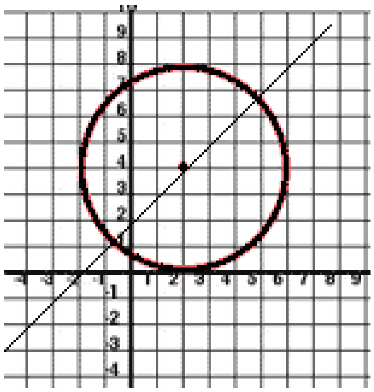
b. $r = 3\cos^2(\theta) + 3\sin^2(\theta)$



10. Consider the graph of $y = A\cos(4x)$ where A is a positive constant.

a) Graph one period **(4 points)**
b) Briefly explain or show how the graph would change if A were a negative constant. **(1 points)**

11. Pictured below is the graphs of a circle and a line. Find the points of intersection (x,y) of the two curves. **(5 points)**



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12. Suppose cos(α) = 1/3 where α is in quadrant IV and sin(β) = 1/2 where β is in Quadrant I.

a) Evaluate: cos(α + β) **(4 points)**
b) Evaluate: sin(2α) **(4 points)**
c) Evaluate: $\sin^2(2\beta) + \cos^2(2\beta)$ **(2 points)**

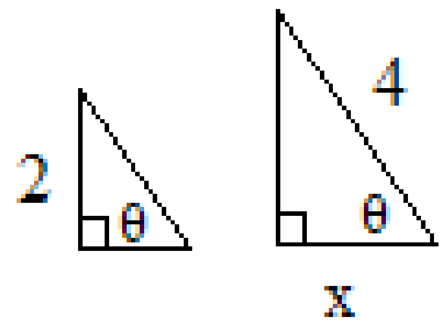
13. Solve the following matrix equation for the unknown matrix X:

AB + 2A = X + 3B, where $A = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 0 \\ 2 & -1 \end{bmatrix}$

14. Give below is a system of linear equations given in the form of an matrix equation alongside the associated augmented matrix. Solve the system of equations using any method: **(8 points)**

$$\begin{bmatrix} 0 & 2 & 3 \\ -2 & 1 & 0 \\ 2 & 0 & -4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix} \text{ which has augmented matrix } \left[\begin{array}{ccc|c} 0 & 2 & 3 & 4 \\ -2 & 1 & 0 & -3 \\ 2 & 0 & -4 & 2 \end{array}\right]$$

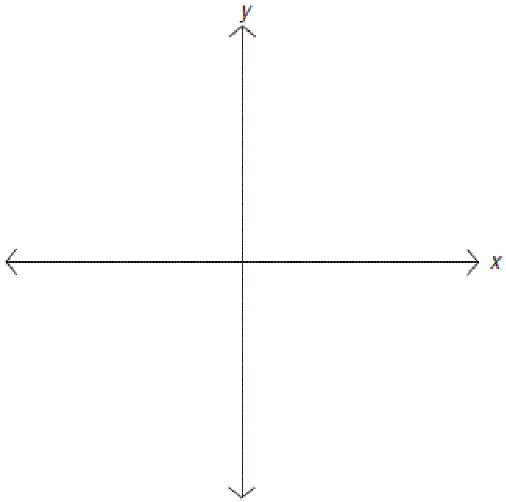
Given two similar triangles, labeled as shown to the right. If $\tan(\theta) = \frac{1}{2}$, find the side length x. **(7 points)**



Solve the following trig equation for all solutions on the range $\theta = [0,2\pi)$

$2\sin(\theta)\cos(2\theta) = \sin(\theta)$

4a. Graph one period of the following equation y=sin(4πx) + 1 **(5 points)**



Fully simplify the following into the form ax^b where 'a' and 'b' are rational numbers.

$$\frac{(8x)^{4/3}\left(\frac{1}{2}x^3\right)^2}{\sqrt[3]{x^{3/2}}}$$