1. Let $X_1, \ldots X_n$ be iid random variables with density

$$f(x;\theta) = \frac{exp\{-(x-\theta)\}}{1+exp\{-(x-\theta)\}}, -\infty < x < \infty, -\infty < \theta < \infty.$$

   (a) Show the existence and uniqueness of maximum likelihood estimator.

   (b) Construct the iterative one-step estimator based on Newton's procedure giving the initial esimator.

2. Let $X_1, \ldots, X_n$ be a random sample from the uniform $(0, \theta]$ distribution.

   (a) Find the complete sufficient statistics for θ and show how to compute the unique minimum variance unbiased estimator of

$$g(\theta) = \frac{e^{t\theta}-1}{t\theta}, t \neq 0,$$

   i.e., the mgf of the distribution.

   (b) Find the close form of the unique minimum variance unbiased estimator of $g(\theta)$.

3. (a) Let X be a *single* observation from the density $f(x;\theta) = (2\theta x + 1 - \theta)I_{[0,1]}(x)$, where $-1 \leq \theta \leq 1$. Find the likelihood ratio test with significance level α of $H_0 : \theta = 0$ versus $H_1 : \theta = 1$.

   (b) If $X_i, \quad i = 1, \ldots, n$, are observations from the Normal distribution with known variance σ_i^2, respectively, and $X_i's$ are mutually independent, construct a test for testing that their means are all equal.