P6 - Rational Exponents and Radicals

Review problems

1. **Operations on integer exponents.** Simplify the following. Assume all variables are positive:

   (a) $\dfrac{(2x^2)y^3}{4x^5y}$ (b) $\dfrac{x^{-2}y^4}{x^5y^{-1}}$

2. **Operations on fractions.** Evaluate and simplify the following:

   (a) $2 + \dfrac{3}{4} \div \dfrac{1}{3}$ (b) $\dfrac{9}{5} \cdot \left(\dfrac{2}{3} - \dfrac{3}{4}\right)$

Basic knowledge

3. Simplify the following. Assume all variables are positive.

   (a) $\dfrac{x^{2/3} \cdot x^{4/5}}{x^{1/2}}$ (b) $\dfrac{(25x^8)^{1/2}}{(4x^6)^{3/2}}$ (c) $(16x^{1/2})^{-5/4}$

   (d) $\sqrt{8x^5} \cdot \sqrt{4x^3}$ (e) $\sqrt[3]{\dfrac{-27x^6}{8y^3}}$ (f) $\sqrt[3]{6x} \cdot \sqrt[3]{9x^2}$

   (g) $3\sqrt{27} - 5\sqrt{12} + \sqrt{75}$ (h) $\sqrt[3]{16} + 5\sqrt[3]{2x^2} - 4\sqrt[3]{4} - 10\sqrt[3]{2000x^2}$

4. Rationalize denominators:

   (a) $\sqrt{\dfrac{3}{5}}$ (b) $\sqrt{\dfrac{4}{3} + 1}$ (c) $\dfrac{3 + \sqrt{2}}{9 - \sqrt{2}}$

Intermediate knowledge

5. Simplify the following. Assume all variables are positive.

   (a) $\sqrt{25x^3\sqrt{16x^4}}$ (b) $\sqrt[3]{\sqrt{x^{24}}}$

6. Rationalize the denominator:

   $\dfrac{\sqrt{2} + \sqrt{3}}{\sqrt{6} - \sqrt{8}}$

Advanced knowledge

7. Rationalize the denominator:

   (a) $\sqrt{\dfrac{3}{\sqrt{3} + \sqrt[3]{5} - 1}}$

8. Convert the given product to a single radical. Assume all variables are positive:

   $\sqrt[5]{x^2} \cdot \sqrt[7]{x^5}$