Information-theoretic Limits for Testing Community Structure of Bipartite Network

Abstract

The bipartite graph is a graph whose nodes set can be divided into two disjoint sets and no two nodes within the same set are adjacent. We denote the number of nodes of two disjoint sets be n_1 and n_2, and denote $n = n_1 + n_2$. W.L.O.G, we assume $n_1 \geq n_2$. Here, we consider the bipartite model to be a random bipartite graph on n nodes with two asymptotically equal cluster sizes. The connection probability of within and between cluster is denoted by δp_n and $(2-\delta)p_n$. In this work, we consider the sparse case $p_n = c\frac{n_1+n_2}{n_1 n_2}$. For the case $n_1 = \alpha n_2$, we prove that the model is mutually contiguous if $c(1-\delta)^2 < \frac{\sqrt{\alpha}}{1+\alpha}$ and they are asymptotically singular if $c(1-\delta)^2 > \frac{\sqrt{\alpha}}{1+\alpha}$. Furthermore, we prove that it is impossible to find an estimate of the labeling of the nodes when $c(1-\delta)^2 < \frac{\sqrt{\alpha}}{1+\alpha}$.