Math 110 Common Final Exam

May 6, 2022

Time: 2 hours and 30 minutes

Instructions: Show all work for full credit.
No outside materials or calculators allowed.

Extra Space: Use the backs of each sheet for extra space. Clearly label when doing so.

Name: key

ID #: ______________________

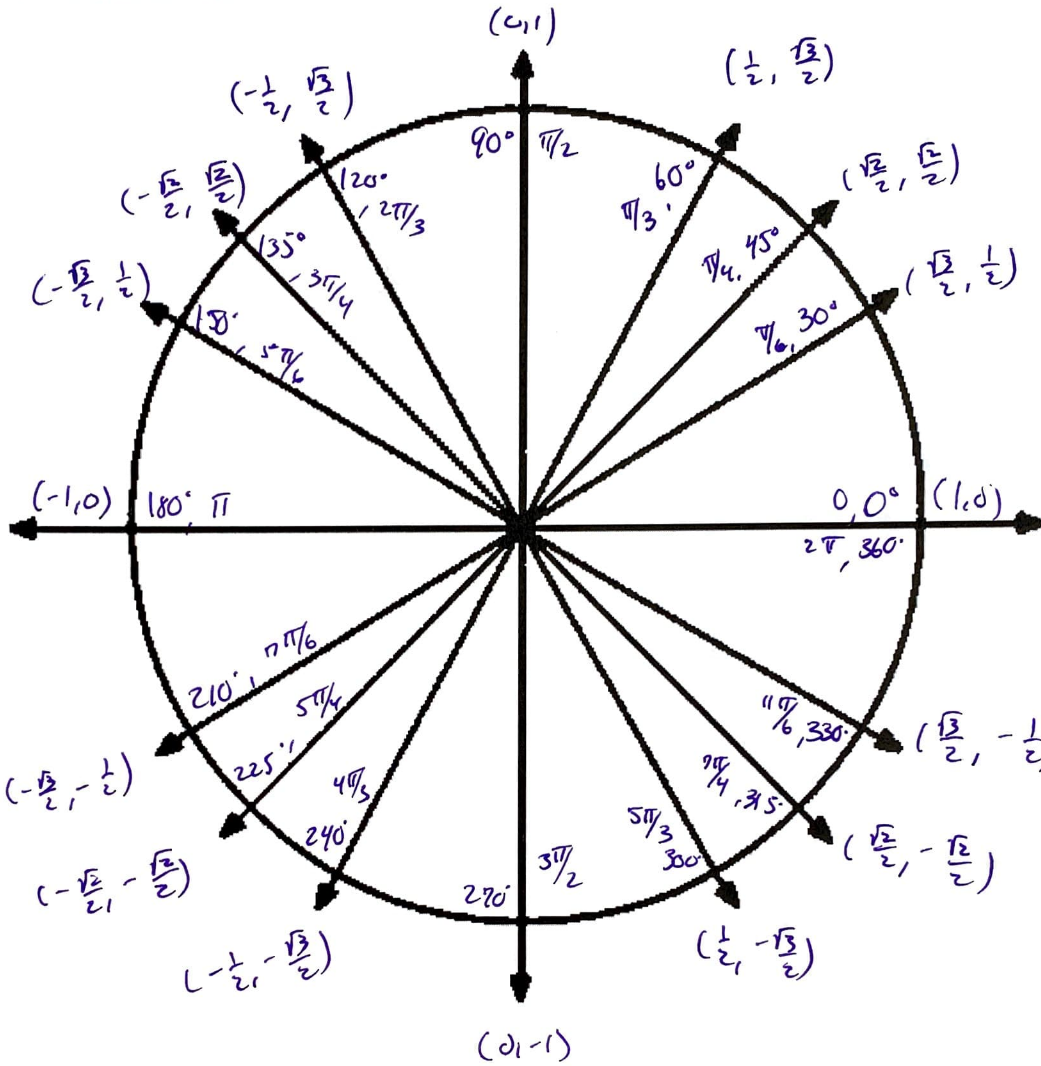
Instructor/Section: ______________

"I pledge by my honor that I have abided by the NJIT Academic Integrity Code."

______________________ (Signature)

Problem		Score
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

1. For all the special angles below, as well as the axes, list (8 points total)
    a. All degrees
    b. All radians
    c. All coordinates



2. Solve the following equations. Be sure to identify any extraneous solutions and if there are no solutions, write "No Solutions" (3 pts each)

a. $\left(\frac{1}{2}\right)^{x} \cdot \left(\frac{1}{8}\right)^{-3x-2} = 32$

$\rightarrow \left(\frac{1}{2}\right)^{x} \cdot \left(\frac{1}{2}\right)^{3(-3x-2)} = \left(\frac{1}{2}\right)^{-5}$

$\rightarrow \log_{1/2} \left(\frac{1}{2}\right)^{x+(-9x-6)} = \log_{1/2} \left(\frac{1}{2}\right)^{-5}$

$\rightarrow x - 9x - 6 = -5 \rightarrow -8x - 6 = -5 \rightarrow -8x = 1$

$\boxed{x = -1/8}$

b. $\log_{10} 1 = \log_{10}(x^2 - 15)$

$1 = x^2 - 15 \rightarrow x^2 = 16 \rightarrow \boxed{x = \pm 4}$

Both are solutions

c. $log_3(4x^2 + 9) - log_3 2 = 2$

$3^{\log_3 \left(\frac{4x^2+9}{2}\right)} = 3^2 \rightarrow \frac{4x^2+9}{2} = 9$

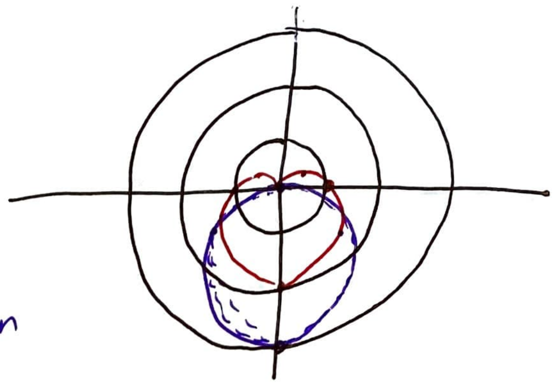
$\rightarrow 4x^2 + 9 = 18 \rightarrow 4x^2 = 9 \rightarrow x^2 = \frac{9}{4} \rightarrow \boxed{x = \pm \frac{3}{2}}$

Both are solutions

3. Graph the polar equations $r = -3sin\theta$ and $r = 1 - sin\theta$ on the same polar graph. Clearly and accurately label any intersection points (full sets of coordinates). (5 points)

θ	$r = -3\sin\theta$	$r = 1 - \sin\theta$
0	0	1
$\pi/6$	$-3/2$	$1/2$
$\pi/2$	-3	0
$5\pi/6$	$-3/2$	$1/2$
π	0	1
$7\pi/6$	$3/2$	$3/2$ *
$3\pi/2$	3	2
$11\pi/6$	$3/2$	$3/2$ *

Intersection



or solving
$$-3\sin\theta = 1 - \sin\theta$$
$$-2\sin\theta = 1$$
$$\sin\theta = \frac{-1}{2}$$
$$\theta = 7\pi/6,\ 11\pi/6$$

Intersection Points:
$(3/2, 7\pi/6)$, $(3/2, 11\pi/6)$

4. Simplify the following using partial fraction decomposition: $\frac{x^3+4x+5}{x^2+3x+2}$ (5 pts)

Improper Fraction → Long Division

$$\begin{array}{r} x - 3 \\ x^2+3x+2 \;\overline{)\; x^3 + 4x + 5} \\ -(x^3 + 3x^2 + 2x) \\ \hline -3x^2 + 2x + 5 \\ -(-3x^2 - 9x - 6) \\ \hline 11x + 11 \end{array}$$

$$\rightarrow \frac{x^3+4x+5}{x^2+3x+2} = x - 3 + \underbrace{\frac{11x+11}{x^2+3x+2}}_{\text{Partial Fract}}$$

$$\rightarrow \frac{11x+11}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$\rightarrow 11x + 11 = A(x+2) + B(x+1)$$

let $x = -2$ → $-11 = -B$ → $\boxed{B = 11}$

let $x = -1$ → $0 = A$ → $\boxed{A = 0}$

$$\therefore \boxed{x - 3 + \frac{11}{x+2}}$$

5. Consider the function $f(x) = \ln(x + 5) + 5$

a. Find the inverse of this function (4 pts)

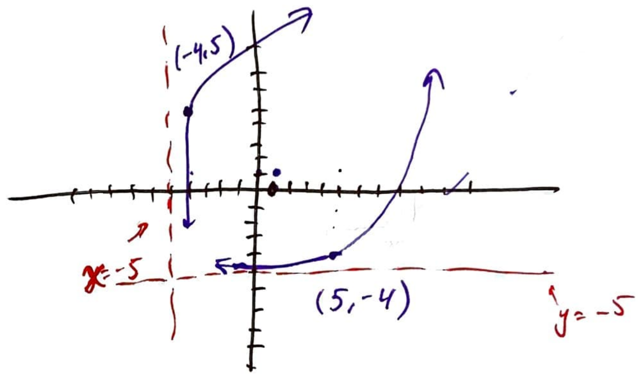
$x = \ln(y+5)+5 \rightarrow e^{x-5} = e^{\ln(y+5)}$

$\rightarrow e^{(x-5)} = y+5 \rightarrow \boxed{y = e^{(x-5)} - 5}$

b. Graph both the original function and it's inverse on the **same coordinate plane.** Be sure to label any asymptotes and at least 1 identifying point for each graph. (3 pts)

$y = \ln(x+5)+5$ Left 5 up 5

$y = e^{(x-5)} - 5$ Right 5 Down 5



6. Given the equation of the ellipse, $16x^2 + y^2 + 96x - 2y + 129 = 0$ identify the coordinates of A) the center, B) the vertices, C) the endpoints of the minor axis, and D) the foci. Write your answers on the appropriate lines below. (2 pts each)

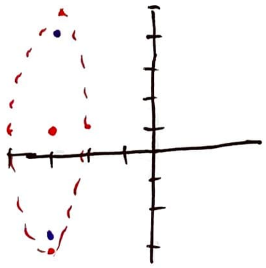
$16x^2 + y^2 + 96x - 2y + 129 = 0$

$16 \cdot 9$

$16(x^2 + 6x + \underline{9}) + (y^2 - 2y + \underline{1}) = -129 + 144 + 1$

$16(x+3)^2 + (y-1)^2 = 16$

$\boxed{(x+3)^2 + \frac{(y-1)^2}{16} = 1}$



$a = 4,\ b = 1$

$c^2 = a^2 - b^2 \Rightarrow c = \pm\sqrt{15}$

A: $(-3, 1)$

B: $(-3, 5), (-3, -3)$

C: $(-4, 1), (-2, 1)$

D: $(-3, 1+\sqrt{15}), (-3, 1-\sqrt{15})$

7. Evaluate the following limits; if the limit doesn't exist, write DNE and **state why**: (3 pts each)

a. $\lim_{x\to-2}\sqrt{4x^2-3}$

$$\lim_{x\to-2}\sqrt{4x^2-3} = \sqrt{4(-2)^2-3} = \sqrt{13}$$

b. $\lim_{x\to1}\frac{x^2+x-2}{x^2-x}$

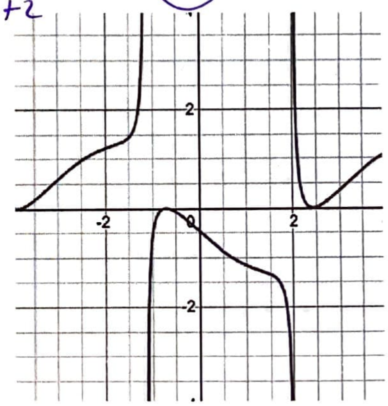
$$\lim_{x\to1}\frac{x^2+x-2}{x^2-x} \to \lim_{x\to1}\frac{(x+2)\cancel{(x-1)}}{x\cancel{(x-1)}}$$

$$\to \lim_{x\to1}\frac{x+2}{x} = \frac{3}{1} = 3$$

c. $\lim_{c\to0}\frac{\sqrt{5c+4}-2}{c}$ $\to \lim_{c\to0}\frac{\sqrt{5c+4}-2}{c}\cdot\frac{\sqrt{5c+4}+2}{\sqrt{5c+4}+2}$

$$\to \lim_{c\to0}\frac{5c\cancel{+4}\cancel{-4}}{\cancel{c}(\sqrt{5c+4}+2)} = \lim_{c\to0}\frac{5}{\sqrt{5c+4}+2} = \frac{5}{4}$$

d. $\lim_{x\to2}\frac{\cos^2(x-4)}{\sin(x-2)}$ Graph pictured here →



DNE ; LHL $\neq$ RHL

8. Consider the function $f(x) = x^3 - 12x$

a. Find an expression for the average rate of change by using the formula $\frac{f(x+h)-f(x)}{h}$ (4 pts)

$f(x+h) \to (x+h)^3 - 12(x+h) \to (x^2+2xh+h^2)(x+h) - 12x - 12h$

$\to x^3 + x^2h + 2x^2h + 2xh^2 + h^2x + h^3 - 12x - 12h$

$\to x^3 + 3x^2h + 3xh^2 + h^3 - 12x - 12h$

$$\frac{f(x+h)-f(x)}{h} = \frac{(\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{12x} - 12h) - (\cancel{x^3} - \cancel{12x})}{h}$$

$$= \frac{\cancel{h}(3x^2 + 3xh + h^2 - 12)}{\cancel{h}} = \boxed{3x^2 + 3xh + h^2 - 12}$$

b. Find an expression for the instantaneous rate of change by taking the limit as h goes to 0 of your answer from part a. (3 pts)

$$\lim_{h \to 0} 3x^2 + 3xh + h^2 - 12 = \boxed{3x^2 - 12}$$

c. Evaluate the instantaneous rate of change (answer from part b) at x = 2 (2 pts)

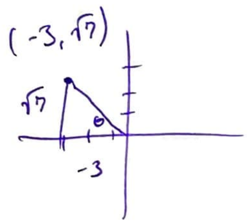
$3(2)^2 - 12 = 0$

9. **Evaluate the following: (3 pts each)**

a. $\sec(-690°)$ $\quad -690+360 = -330+360 = 30$

$\rightarrow \boxed{\sec 30 = \frac{2}{\sqrt{3}}}$

b. Sine, if the point on the terminal side of the angle θ is $(-3, \sqrt{7})$



$c^2 = (\sqrt{7})^2 + (-3)^2 = 7+9 = 16 \quad \therefore c = 4$

$\boxed{\sin\theta = \frac{\sqrt{7}}{4}}$

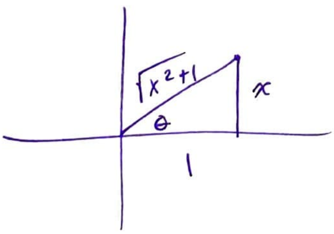
c. $tan^{-1}\left(\cot\left(\frac{5\pi}{6}\right)\right)$ $\quad \cot\left(\frac{5\pi}{6}\right) = -\sqrt{3}$

$\tan^{-1}(-\sqrt{3}) = -\pi/3$

d. $\sin(\arctan(x))$ in the first quadrant

$\arctan x = \theta$

$\rightarrow \tan\theta = x = \frac{x}{1}$



$\therefore hyp = \sqrt{x^2+1}$

$\sin(\arctan x) = \sin\theta = \frac{x}{\sqrt{x^2+1}}$

10. Solve the following for all solutions belonging to the interval $[0, 2\pi)$ (4 pts each)

a. $\sin\left(x+\frac{\pi}{4}\right)+\sin\left(x-\frac{\pi}{4}\right)=-1$

$\sin x\cos\frac{\pi}{4}+\cancel{\sin\frac{\pi}{4}\cos x}+\sin x\cos \pi/4-\cancel{\sin \pi/4 \cos x}=-1$

$2\sin x\cos \pi/4=-1 \rightarrow 2\sin x\left(\frac{\sqrt{2}}{2}\right)=-1$

$\sin x=-\frac{1}{\sqrt{2}} \Rightarrow \sin x=-\frac{\sqrt{2}}{2}$ $\boxed{x=5\pi/4,\ 7\pi/4}$

b. $0=3sin\theta+2-cos^2\theta+sin^2\theta$

$3\sin\theta-(1-\sin^2\theta)+\sin^2\theta=-2$

$3\sin\theta-1+\sin^2\theta+\sin^2\theta=-2$

$2\sin^2\theta+3\sin\theta+1=0$

$(2\sin\theta+1)(\sin\theta+1)=0$

$\sin\theta=-\frac{1}{2}$, $\sin\theta=-1$

$\boxed{\theta=7\pi/6,\ 11\pi/6 \qquad \theta=3\pi/2}$

c. $-cos3x=cos5x$

$\cos 5x+\cos 3x=0 \rightarrow 2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)=0$

$\rightarrow 2\cos(4x)\cos x=0 \rightarrow (\cos 4x)(\cos x)=0$

For $\cos x=0 \rightarrow \boxed{x=\pi/2,\ 3\pi/2}$

For $\cos 4x=0 \rightarrow 4x=\pi/2+n\pi$

$\rightarrow x=\frac{\pi}{8}+\frac{n\pi}{4}$

$n=0 \rightarrow \pi/8$

$n=1 \rightarrow \pi/8+2\pi/8=3\pi/8$

$n=2 \rightarrow \pi/8+4\pi/8=5\pi/8$

$n=3 \rightarrow \pi/8+6\pi/8=7\pi/8$

$n=4 \rightarrow \pi/8+8\pi/8=9\pi/8$

$n=5 \rightarrow \pi/8+10\pi/8=11\pi/8$

$n=6 \rightarrow \pi/8+12\pi/8=13\pi/8$

$n=7 \rightarrow \pi/8+14\pi/8=15\pi/8$

d. $cos2\theta=cos^2\theta$

$2\cos^2\theta-1=\cos^2\theta$

$\cos^2\theta=1$

$\cos\theta=\pm 1$, $\theta=0, \pi$

11. Three unique numbers sum to 108. The smallest is half the size of the largest, while the middle is three quarters the size of the largest. <u>Set up a system of three equations</u> to solve for the three values. (5 pts)

x = small, y = middle, z = largest $\rightarrow$ $x = \frac{z}{2} \rightarrow 2x = z$

$\rightarrow y = \frac{3}{4}z \rightarrow \frac{4}{3}y = z$

$$\begin{cases} ① \ x + y + z = 108 \\ ② \ 2x - z = 0 \\ ③ \ \frac{4}{3}y - z = 0 \end{cases}$$

①+② $x + y + z = 108$

$2x - z = 0$

④ $3x + y = 108$

,

①+③ $x + y + z = 108$

$\frac{4}{3}y - z = 0$

⑤ $x + \frac{7}{3}y = 108$

,

④ − 3·⑤

$3x + y = 108$

$-(3x + 7y = 324)$

$-6y = -216$

$\therefore \boxed{y = 36}$

$z = \frac{4}{3}y \rightarrow$

$z = \frac{4}{3}(36)$

$\boxed{z = 48}$

$2x = z$

$2x = 48$

$\boxed{x = 24}$

$\therefore$ Solution:

$\boxed{(24, 36, 48)}$

12. For the following statements, write True of False; no work required: (1 pts each)

a. The range of $y = sin3x$ is [-3 , 3]

False $\rightarrow$ Range is $[-1, 1]$

b. If $r = 5\ feet\ and\ \theta = 30°$ then $s = 5 \cdot 30 = 150 feet$

False $\rightarrow$ θ needs to be in radians

c. $sin75° = sin45° + sin30°$

False $\rightarrow$ $\sin 75 = \sin(45 + 30) = \sin 45 \cos 30 + \sin 30 \cos 45$

d. The equation $\sec x = \frac{1}{2}$ has 2 solutions in $[0 , 2\pi)$

False $\rightarrow$ $\sec x = \frac{1}{2} \Rightarrow \cos x = 2$ $\therefore$ No solutions!