Math 110 Common Exam #3

April 20, 2022

Time: 1 hour and 25 minutes

Instructions: Show all work for full credit.
No outside materials or calculators allowed.

Extra Space: Use the backs of each sheet for extra space. Clearly label when doing so.

Name: Key

ID #: ______

Instructor/Section: ______

"I pledge by my honor that I have abided by the NJIT Academic Integrity Code."

______ (Signature)

Problem		Score
1		
2		
3		
4		
5		
6		
7		
8		
9		

1. **Evaluate the following: (5 pts each)**

   a. $\sin 75° \cdot \sin 15°$

$\rightarrow \sin x \cdot \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y))$

$\Rightarrow \frac{1}{2}(\cos(75-15) - \cos(75+15)) = \frac{1}{2}(\cos 60 - \cos 90)$

$= \frac{1}{2}\left(\frac{1}{2}\right) = \boxed{\frac{1}{4}}$

   b. $\sin 435° - \sin 165°$

$\rightarrow \sin x - \sin y = 2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)$

$\Rightarrow 2\sin\left(\frac{435-165}{2}\right)\cos\left(\frac{435+165}{2}\right) = 2\sin\left(\frac{270}{2}\right)\cos\left(\frac{600}{2}\right)$

$= 2\sin(135)\cos(300) = 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \boxed{\frac{\sqrt{2}}{2}}$

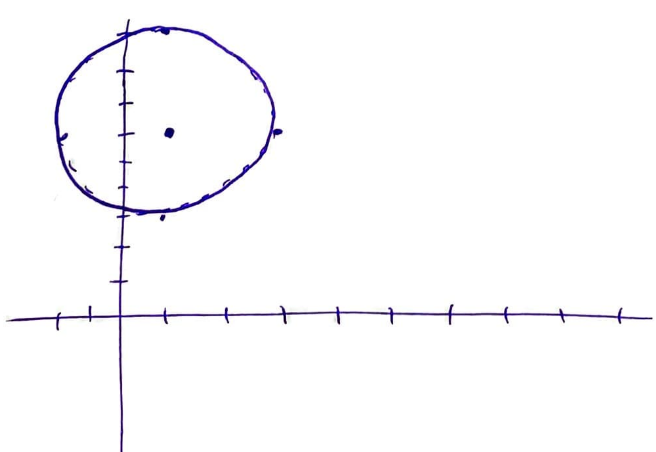
2. **Sketch a graph of the following: (5 pts)**

$$-12y + y^2 - 2x = -28 - x^2$$

$x^2 - 2x + y^2 - 12y = -28$

$\rightarrow x^2 - 2x + \underline{1} + y^2 - 12y + \underline{36} = -28 + 1 + 36$

$(x-1)^2 + (y-6)^2 = 9$ Circle: center $(1, 6)$
radius 3



3. **Solve the following trigonometric equations for solutions in the interval $[0, 2\pi)$ (6 pts each)**

a. $-sin\frac{\theta}{2} - cos\theta = -1$

Method 1

$-\sin\frac{\theta}{2} - \cos\theta = -1 \rightarrow \left[-\left(\pm\sqrt{\frac{1+\cos\theta}{2}}\right) = \cos\theta - 1\right]^2$

$\rightarrow \frac{1+\cos\theta}{2} = \cos^2\theta - 2\cos\theta + 1$

$\rightarrow 2\cos^2\theta - 3\cos\theta + 1 = 0$

$\rightarrow (2\cos\theta - 1)(\cos\theta - 1) = 0 \rightarrow \cos\theta = \frac{1}{2}$

$\cos\theta = 1$

$\boxed{\therefore\ \theta = \pi/3,\ 5\pi/3,\ 0}$

Method 2

Let $x = \frac{\theta}{2}$

$\rightarrow -\sin x - \cos 2x = -1$

$\rightarrow -\sin x - (1 - 2\sin^2 x) = -1$

$\rightarrow -\sin x - \cancel{1} + 2\sin^2 x = -\cancel{1}$

$\rightarrow \sin x(-1 + 2\sin x) = 0$

$\sin x = 0$, $2\sin x - 1 = 0$

$x = 0$, $\sin x = \frac{1}{2} \rightarrow x = \pi/6,\ 5\pi/6$

$\frac{\theta}{2} = 0$

$\boxed{\theta = 0}$, $\boxed{\theta = \pi/3,\ 5\pi/3}$

b. $2\sin\left(x - \frac{\pi}{4}\right) - 3 = -5$

Let $x - \frac{\pi}{4} = \theta$

$\rightarrow 2\sin\theta - 3 = -5 \rightarrow 2\sin\theta = -2$

$\rightarrow \sin\theta = -1 \qquad \theta = \frac{3\pi}{2}$

$x - \frac{\pi}{4} = \frac{3\pi}{2} \rightarrow x = \frac{3\pi}{2} + \frac{\pi}{4} = \boxed{\frac{7\pi}{4}}$

c. $\sqrt{3}\sec(2x) + 2 = 0$

Let $\theta = 2x$

$\sqrt{3}\sec\theta + 2 = 0 \rightarrow \sec\theta = \frac{-2}{\sqrt{3}}$

$\rightarrow \cos\theta = -\frac{\sqrt{3}}{2}$, $\theta = \frac{5\pi}{6}, \frac{7\pi}{6} \rightarrow 2x = \frac{5\pi}{6} + 2n\pi$

$2x = \frac{7\pi}{6} + 2n\pi$

$\rightarrow x = \frac{5\pi}{12} + n\pi$

$x = \frac{7\pi}{12} + n\pi$

$n = 0 \rightarrow 5\pi/12,\ 7\pi/12$

$n = 1 \rightarrow 17\pi/12,\ 19\pi/12$

4. Solve the following trigonometric equations for all possible solutions (5 pts each)

a. $3\cancel{\cos\theta} = \sqrt{2}\tan\theta\cos\theta + \tan\theta + 3\cancel{\cos\theta}$

$\sqrt{2}\tan\theta\cos\theta + \tan\theta = 0$

$\rightarrow \tan\theta(\sqrt{2}\cos\theta + 1) = 0$

$\tan\theta = 0$, $\cos\theta = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\theta = \boxed{n\pi}$ $\theta = \boxed{\begin{matrix}\frac{3\pi}{4} + 2n\pi \\ \frac{5\pi}{4} + 2n\pi\end{matrix}}$

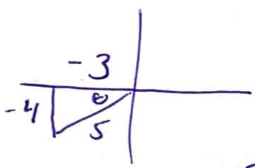
b. $2\cos x + 2 = \sin^2 x$

$2\cos x + 2 = 1 - \cos^2 x$

$\rightarrow \cos^2 x + 2\cos x + 1 = 0 \rightarrow (\cos x + 1)(\cos x + 1) = 0$

$\cos x = -1$ $\boxed{x = \pi + 2n\pi}$

5. Given that $\sin\theta = -\frac{4}{5}, \pi \le \theta < \frac{3\pi}{2}$, find the following: (5 pts each)



a. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$, $\tan\theta = \frac{4}{3}$

$= \frac{2(\frac{4}{3})}{1-(\frac{4}{3})^2} = \frac{8/3}{1-16/9} = \frac{8/3}{-7/9} = \frac{8}{\cancel{3}} \cdot (-\frac{\cancel{9}^{3}}{7}) = \boxed{-\frac{24}{7}}$

b. $\cos^2\theta$ Method 1 $\rightarrow \cos\theta = -\frac{3}{5}$ $\therefore \boxed{\cos^2\theta = \frac{9}{25}}$

Method 2: $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ $\rightarrow \cos 2\theta = 1 - 2\sin^2\theta$

$1 - 2(-\frac{4}{5})^2$

$1 - \frac{32}{25} = -\frac{7}{25}$

$= \frac{1-\frac{7}{25}}{2} = \frac{18/25}{2} = \frac{18}{25}\cdot\frac{1}{2} = \boxed{\frac{9}{25}}$

6. Convert the following: (3 pts each)

a. coordinates from Polar to rectangular

i. $\left(-4, \frac{3\pi}{2}\right)$

$x = -4\cos\frac{3\pi}{2} = 0$

$y = -4\sin\frac{3\pi}{2} = 4$

$\therefore (0, 4)$

ii. $(-2, 150°)$

$x = -2\cos 150° = \sqrt{3}$

$y = -2\sin 150° = -1$

$\therefore (\sqrt{3}, -1)$

iii. $(-2, \pi)$

$x = -2\cos\pi$

$y = -2\sin\pi$

$\therefore (2, 0)$

b. coordinates from Rectangular to polar, where $r \geq 0, \theta$ *is between* $[0, 2\pi)$(3 pts each)

i. $(\sqrt{3}, 1)$ in Q_1

$r = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \pm 2$, $r = 2$

$\tan\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\therefore \theta = \pi/6$

$\boxed{\therefore (2, \pi/6)}$

ii. $(-2\sqrt{2}, -2\sqrt{2})$ in Q 3

$r = \sqrt{(-2\sqrt{2})^2 + (-2\sqrt{2})^2} = \sqrt{8+8} = \pm 4 \to r = 4$

$\tan\theta = \frac{-2\sqrt{2}}{-2\sqrt{2}} = 1$ $\therefore \theta = \frac{5\pi}{4}$ (in Q_3)

$\boxed{\therefore (4, \frac{5\pi}{4})}$

c. Equations from polar to rectangular, and identify the graph (shape) of the equation along with any intercepts, slopes, centers, radii, etc. (4 pts each)

i. $tan\theta = 2$ $\rightarrow \frac{y}{x} = 2 \rightarrow y = 2x$

Line, slope 2, intercept 0

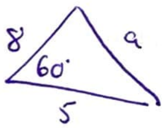
ii. $\left[r = -4\cos\theta - 4\sin\theta \right] r \rightarrow r^2 = -4r\cos\theta - 4r\sin\theta$

$\rightarrow x^2 + y^2 = -4x - 4y \rightarrow x^2 + 4x + \underline{4} + y^2 + 4y + \underline{4} = 0 + 4 + 4$

$\boxed{(x+2)^2 + (y+2)^2 = 8 \Rightarrow \text{circle; center}(-2,-2) \quad \text{rad} = 2\sqrt{2}}$

7. Suppose a triangle has side lengths, 5 and 8, with the angle between them measuring 60°.

a. Find the measure of the missing side. (5 pts)



Law of Cos $\rightarrow a^2 = b^2 + c^2 - 2bc\cos A$

$= 5^2 + 8^2 - 2(5)(8)\cos 60$

$= 25 + 64 - 80\left(\frac{1}{2}\right)$

$= 89 - 40 =$

$= 49$

$\therefore \boxed{a = 7}$

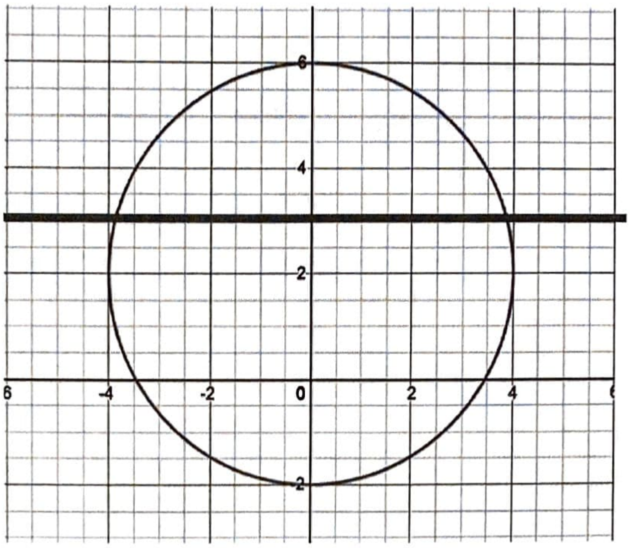
b. Find the area of the triangle (4 pts)

$k = \frac{1}{2} bc \sin A$

$= \frac{1}{2}(5)(8)\sin 60$

$= 20 \cdot \left(\frac{\sqrt{3}}{2}\right) = \boxed{10\sqrt{3}}$

8. Find the coordinates of the intersection points for the circle and line pictured. (5 pts)



Line: $y = 3$

Circle: $x^2 + (y-2)^2 = 16$ $\rightarrow$ Plug in $y = 3 \rightarrow x^2 + (3-2)^2 = 16$

$\rightarrow x^2 + 1 = 16 \rightarrow x^2 = 15 \rightarrow x = \pm\sqrt{15} \Rightarrow$ Coords: $(\sqrt{15}, 3)$ $(-\sqrt{15}, 3)$

9. For each type of oblique triangle below identify which procedure is appropriate for solving the triangle: (write "Law of Sines" or "Law of Cosines" or "Both") (2 pts each)

a. SAS Law of Cosines

b. AAS Law of Sines

c. SSS Law of Cosines

d. ASA Law of Sines

e. SSA Both