Math 110 Common Exam #2

March 9, 2022

Time: 1 hour and 25 minutes

Instructions: Show all work for full credit.
No outside materials or calculators allowed.

Extra Space: Use the backs of each sheet for extra space. Clearly label when doing so.

Name: Key

ID #: _____

Instructor/Section: _____

"I pledge by my honor that I have abided by the NJIT Academic Integrity Code."

_____ (Signature)

Problem		Score
1		
2		
3		
4		
5		
6		
7		
8		
9		

1. Find the reference angle for the following: (3 pts each)

a. 215° $215 \in Q3$

$\therefore \theta' = \theta - 180$

$\rightarrow 215 - 180 = \boxed{35°}$

b. −200° $-200 \in Q2$

$\therefore \theta' = 180 - \theta$

$= 180 - (-200) = 380$, coterminal with $\boxed{20°}$

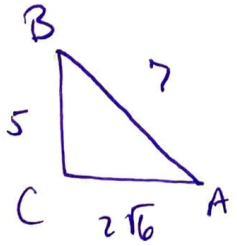
or $\rightarrow -200°$ coterminal with $160°$

$\theta' = 180 - 160 = \boxed{20°}$

c. $\frac{43\pi}{18} - \frac{36\pi}{18} = \frac{7\pi}{18} \in Q_1$

$\therefore \theta' = \theta \rightarrow \theta' = \frac{7\pi}{18}$

2. In a right triangle ABC (where C is the right angle), find $csc\ A\ if\ a = 5\ and\ b = 2\sqrt{6}$ (6 pts)



$\therefore \csc A = \frac{hyp}{opp} = \boxed{\frac{7}{5}}$

$c = \sqrt{5^2 + (2\sqrt{6})^2}$

$= \sqrt{25 + 24}$

$= \sqrt{49}$

$= 7$

3. Find the exact value of the following or write "undefined" if no value exists: (3 pts each for a – c, 6 pts each for d – f)

a. $\csc 780°$

$780 - 2(360) = 780 - 720 = 60$

$\csc 60 = \boxed{\frac{2}{\sqrt{3}} \text{ or } \frac{2\sqrt{3}}{3}}$

b. $\sin\frac{23\pi}{4}$

$\frac{23\pi}{4} - 2\left(\frac{8\pi}{4}\right) = \frac{23\pi}{4} - \frac{16\pi}{4} = \frac{7\pi}{4}$

$\sin\frac{7\pi}{4} = \boxed{-\frac{\sqrt{2}}{2}}$

c. $\sec -\frac{5\pi}{2}$

$-\frac{5\pi}{2} + 2\left(\frac{4\pi}{2}\right) = -\frac{5\pi}{2} + \frac{8\pi}{2} = \frac{3\pi}{2}$

$\sec\frac{3\pi}{2}$ $\boxed{\text{undefined}}$

d. $\tan 75°$

$\tan(30+45) = \frac{\tan 30 + \tan 45}{1 - \tan 30 \tan 45} = \frac{\frac{\sqrt{3}}{3} + 1}{1 - \left(\frac{\sqrt{3}}{3} \cdot 1\right)}$

$= \frac{\frac{\sqrt{3}+3}{3}}{\frac{3-\sqrt{3}}{3}} = \boxed{\frac{\sqrt{3}+3}{3-\sqrt{3}}}$

e. $\cos\left(-\frac{\pi}{12}\right)$ $= \cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

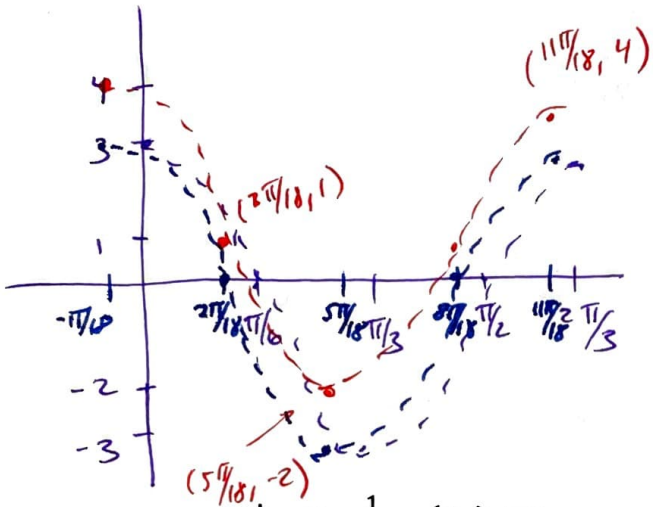
f. $\sin\frac{\pi}{9}\cos\frac{13\pi}{18} + \cos\frac{\pi}{9}\sin\frac{13\pi}{18}$ $= \sin\left(\frac{\pi}{9} + \frac{13\pi}{18}\right)$

$= \sin\left(\frac{15\pi}{18}\right) = \sin\left(\frac{5\pi}{6}\right) = \boxed{\frac{1}{2}}$

4. Graph at least 1 period the following; be sure to label at least 2 identifying points: (6 pts each)

a. $y = 1 + 3\cos\left(3\theta + \frac{\pi}{6}\right)$ → $3\cos\left[3\left(\theta + \frac{\pi}{18}\right)\right] + 1$

amp: 3
period $\frac{2\pi}{3}$
Left $\pi/18$
up 1

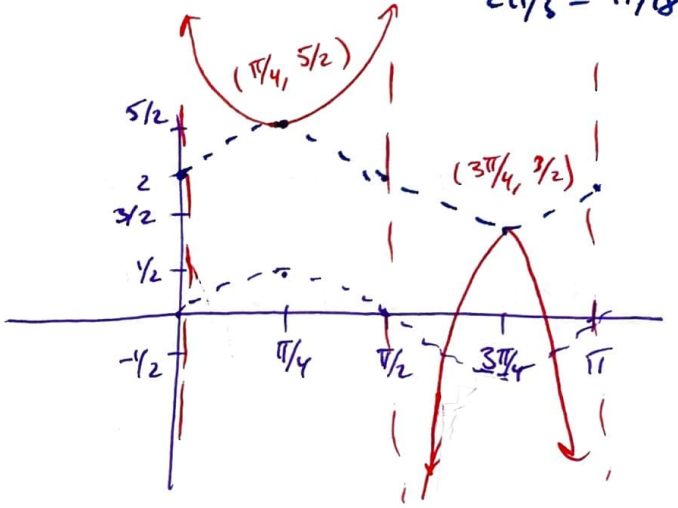


Left: Every point $-\pi/18$

$0 - \pi/18 = -\pi/18$
$\pi/6 - \pi/18 = 2\pi/18$
$\pi/3 - \pi/18 = 5\pi/18$
$\pi/2 - \pi/18 = 8\pi/18$
$2\pi/3 - \pi/18 = 11\pi/18$

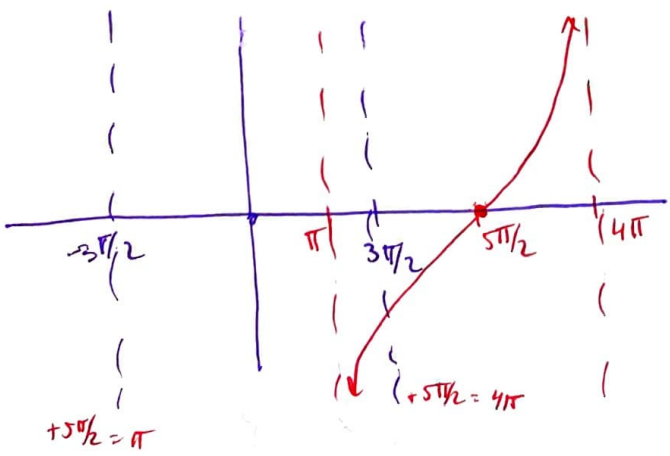
b. $y = \frac{1}{2}\csc(2x) + 2$

amp: $\frac{1}{2}$
period: $\frac{2\pi}{2} = \pi$
up: 2

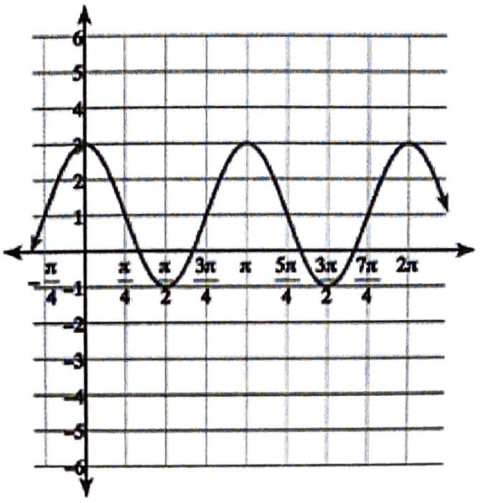


c. $y = \tan\left(\frac{x}{3} - \frac{5\pi}{6}\right)$ → $\tan\left[\frac{1}{3}\left(x - \frac{5\pi}{2}\right)\right]$

period: $\frac{\pi}{1/3} = 3\pi$
right: $5\pi/2$



5. Given the graph below, find an equation in either sine or cosine. (5 pts)



Cos → up 1, amp 2, period $= \pi \Rightarrow \frac{2\pi}{b} = \pi \rightarrow b = 2$

$\therefore y = 2\cos(2x) + 1$

Sin → up 1, amp 2 period $\pi \rightarrow b = 2$, shift left $\pi/4$

$\therefore y = 2\sin[2(x + \pi/4] + 1$ or $2\sin(2x + \pi/2) + 1$

6. If $\csc\theta = 2, find \sec\left(\frac{\pi}{2} - \theta\right)$ 5 pts

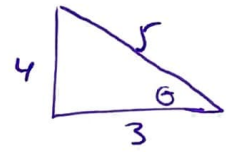
Cofunction identity: $\sec(\pi/2 - \theta) = \csc\theta$

$\therefore \sec(\pi/2 - \theta) = \boxed{2}$

7. Evaluate the following: (5 pts each)

a. $\csc\left(\tan^{-1}\left(\frac{4}{3}\right)\right)$

Let $\theta = \tan^{-1}\left(\frac{4}{3}\right)$



$\therefore \csc\theta = \frac{5}{4}$

b. $\arccos\left(\cot\frac{\pi}{4}\right)$

$\arccos(1) = 0$

8. A merry-go-round makes 8 revolutions per minute. (4 pts each)

a. What is the angular speed of the merry-go-round?

$8\ \frac{rev}{min} \cdot \frac{2\pi\ rad}{1\ rev} = \boxed{16\pi\ rad/min}$

b. What is the linear speed of a horse that is 12 feet from the center?

$v = r\omega \rightarrow v = 12(16\pi) = \boxed{192\pi\ ft/min}$

9. Verify the identities: (6 pts each)

a. $\frac{tan^2x}{sin^2x} = tan^2x + 1$

$$\frac{\sin^2x / \cos^2x}{\sin^2x / 1}$$

$$\rightarrow \frac{\sin^2x}{\cos^2x} \cdot \frac{1}{\sin^2x}$$

$$\rightarrow \frac{1}{\cos^2x}$$

$$\rightarrow \sec^2x = \tan^2x + 1$$

$$\rightarrow \sec^2x = \sec^2x \quad \checkmark$$

b. $\cos(\theta + 90°) = -\sin\theta$

$$\rightarrow \cos\theta\cos 90^\circ - \sin\theta\sin 90$$

$$\rightarrow -\sin\theta(1)$$

$$\rightarrow -\sin\theta = -\sin\theta \quad \checkmark$$