## Ph.D. Prelim: Exam B

## **Statistical Inference**

## May, 2024

1. Let  $X_1, \ldots, X_n$  be independent identically distributed random variables with pdf (probability density function)

$$f(x) = \frac{1}{\lambda} \exp(-\frac{x}{\lambda})$$

where x and  $\lambda$  are both positive. Find the uniformly minimum variance unbiased estimator (UMVUE) of  $\lambda^2$ .

- 2. Suppose that  $X_1, \ldots, X_n$  are iid Bernoulli(p) where  $n \ge 2$  and 0 is the unknown parameter.
  - (a) Derive the UMVUE of p(1-p) and show that it is a consistent estimator.
  - (b) Find the Cramer-Rao lower bound for estimating p(1-p).
- 3. Let  $X_1, \ldots, X_n$  be independent identically distributed random variables from a distribution with pdf

$$f(x) = \frac{2}{\sigma\sqrt{2\pi}} \frac{1}{x} \exp\left(\frac{-[\log(x)]^2}{2\sigma^2}\right)$$

where  $\sigma > 0$  and  $x \ge 1$ .

- (a) What is the UMP (uniformly most powerful) level  $\alpha$  test for  $H_0: \sigma = 1$  vs.  $H_1: \sigma = 2$ ?
- (b) Find the UMP (uniformly most powerful) level  $\alpha$  test for  $H_0: \sigma = 1$  vs.  $H_1: \sigma > 1$  and derive the critical value of the test.