Ph.D. Qualifying Exam B

Statistical Inference

May 22, 2023

1. **(20 points)** Suppose that $Y_1, \ldots, Y_n$ is a random sample from a Bernoulli distribution with success probability p. Let $\tau(p) = p(1-p)$. Find the MVUE of $\tau(p)$.

2. **(20 points)** Let $Y_1, \ldots, Y_n$ be a random sample from a population with density function

$$f(y;\theta) = \begin{cases} \theta y^{\theta-1}, & \text{if } 0 \le y \le 1, \theta > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) Find a uniformly most powerful test of size α for testing $H_0 : \theta = 1$ against $H_1 : \theta > 1$. Specify your best critical region in terms of a test statistic whose null distribution is clearly identifiable. Justify why the test is uniformly most powerful for all alternatives $\theta > 1$.

   (b) Use part (2a) to find a uniformly most powerful, size $\alpha = 0.05$, test when $n = 1$.

3. **(20 points)** Let X have a Poisson distribution with parameter θ. Define $T = I(X = 0)$, where $I(X = 0) = 1$ if $X = 0$ and is 0 otherwise.

   (a) Find the Rao–Cramér lower bound for (estimating the variance of) T and show that it is strictly less than the variance of T.

   (b) Show that T is an MVUE of its expectation.