NJIT New Jersey's Science & Technology University

THE COLLEGE OF SCIENCE AND LIBERAL ARTS

THE DEPARTMENT OF MATHEMATICAL SCIENCES

MATH 707: Optimization – convex analysis and continuous optimization

Spring 2021 Graduate Course Syllabus

NJIT Academic Integrity Code: All Students should be aware that the Department of Mathematical Sciences takes the University Code on Academic Integrity at NJIT very seriously and enforces it strictly. This means that there must not be any forms of plagiarism, i.e., copying of homework, class projects, or lab assignments, or any form of cheating in quizzes and exams. Under the University Code on Academic Integrity, students are obligated to report any such activities to the Instructor.

COURSE INFORMATION

Course Description: This graduate course covers modern topics in continuous optimization. Starting with theoretical foundations, the course develops a working knowledge of state-of-the-art algorithms, and concludes with applications in engineering and the applied sciences.

Math 707–004 covers three key areas:
1. Developing a rigorous treatment of convex analysis (include conditions for optimality). Convex analysis forms the mathematical underpinnings for much of modern optimization.
2. A systematic development of interior-point methods. Interior-point methods are state-of-the-art algorithms for solving (continuous) optimization problems, such as those that arise in physical applications of applied mathematics.
3. Exposure to modern applications of optimization in applied mathematics, including obstacle problems, image processing, compressed sensing of signals, stochastic gradient descent and Tikhonov regularization using norms.

Number of Credits: 3

Prerequisites: This course draws extensively on linear algebra (Math 631) and multivariable calculus (Math 213). The course makes use of some undergraduate analysis (Math 480-481) and numerical methods (Math 613-614). Students may also seek permission from the instructor for enrollment.

Course-Section and Instructors

Course-Section	Instructor
Math 707—004	rofessor D. Shirokoff

Office Hours for All Math Instructors: Spring 2021 Office Hours and Emails

Textbook: There is no required textbook for this class – we will use resources from several textbooks (the primary ones listed below). Please note that the materials listed below are freely available.

Primary Textbook:

Title	Convex Optimization

Author	Boyd and L. Vandenberghe
Edition	1st Ed.
Publisher	Cambridge University Press
ISBN #	978-0-521-83378-3
Website	Textbook is freely available at the authors website: https://web.stanford.edu/~boyd/cvxbook/

Secondary Textbook:

Title	Convex Optimization Theory
Author	D. P. Bertsekas
Edition	1st Ed.
Publisher	Athena Scientific
ISBN #	978-1-521-886529-31-1
Website	http://www.athenasc.com/convexduality.html The author (D.P. Bertsekas) has made course notes (that cover the same material as the textbook) freely available on his website.

University-wide Withdrawal Date:The last day to withdraw with a **W** is **Monday, April 5, 2021**. It will be strictly enforced.

POLICIES

DMS Course Policies: All DMS students must familiarize themselves with, and adhere to, the Department of Mathematical Sciences Course Policies, in addition to official university-wide policies. DMS takes these policies very seriously and enforces them strictly.

Grading Policy: The final grade in this course will be determined as follows:

Homework Assignments	50%
Exam	20%
Project/Presentations	30%

Your final letter grade will be based on the following tentative curve.

A	85 - 100	**C**	65 - 69
B+	80 - 84	**D**	50 - 64
B	75 - 79	**F**	0 - 49
C+	70 - 74		

Attendance Policy: Attendance at all classes will be recorded and is **mandatory**. Please make sure you read and fully understand the Math Department's Attendance Policy. This policy will be strictly enforced. AttendanceNote

Course Policies: See course website.

The final exam will test your knowledge of all the course material taught in the entire course. Make sure you read and fully understand the Math Department's Examination Policy. This policy will be strictly enforced.

Makeup Exam Policy: To properly report your absence from a midterm or final exam, please review and follow the required steps under the DMS Examination Policy found here:

- http://math.njit.edu/students/policies_exam.php

Cellular Phones: All cellular phones and other electronic devices must be switched off during all class times.

ADDITIONAL RESOURCES

Accommodation of Disabilities: The Office of Accessibility Resources and Services (OARS) offers long term and temporary accommodations for undergraduate, graduate and visiting students at NJIT.

If you are in need of accommodations due to a disability please contact Chantonette Lyles, Associate Director of Office of Accessibility Resources and Services at 973-596-5417 or via email at lyles@njit.edu. The office is located in Kupfrian Hall, Room 201. A Letter of Accommodation Eligibility from the Office of Accessibility Resources and Services authorizing your accommodations will be required.

For further information regarding self identification, the submission of medical documentation and additional support services provided please visit the Office of Accessibility Resources and Services (OARS) website at:

- https://www.njit.edu/studentsuccess/accessibility/

Important Dates (See: Spring 2021 Academic Calendar, Registrar)

Date	Day	Event
January 19, 2021	T	First Day of Classes
January 23, 2021	S	Saturday Classes Begin
January 25, 2021	M	Last Day to Add/Drop Classes
March 14 - March 21, 2021	Su - Su	Spring Recess - No Classes
April, 2, 2021	F	Good Friday - No Classes
April 5, 2021	M	Last Day to Witdraw
May 4, 2021	T	Friday Classes Meet
May 4, 2021	T	Last Day of Classes
May 5 & May 6, 2021	W & R	Reading Days
May 7 - May 13, 2021	F - R	Final Exam Period

Course Outline

Section/Chapter	Topic	Number of Lectures
1	**Introduction to convex analysis:** definitions and properties of convex sets and (strictly) convex functions. Caratheodory's theorem. Norms are convex functions, both finite and infinite dimensional. L2 versus L1 norms. Separating hyperplanes. Convex envelopes.	6
2	**Conditions for optimality**: Development of the (necessary) KKT conditions for	4

	optimality in convex and non-convex problems from a geometric viewpoint. Discussion on when KKT conditions are sufficient. Includes both equality (Lagrange multipliers) and inequality constraints.	
3	**Introduction to Numerical Algorithms:** Interior point methods as a way to handle inequality constraints. Discussion of Newton's method and gradient descent as approaches for solving the interior point (penalty) equations. Examples from PDE based problems, including Stokes equations, obstacle problems, elasticity, image processing.	4
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5	**Focus on Special Topics. Possible topics include:** (i) Optimal control problems/PDE constrained optimization, (ii) Stochastic gradient descent, (iii) Obstacle problems, (iv) Optimal transport, (v) Compressed sensing, (vi) Semi-definite programing, (vii) Image processing.	2 per topic

GENERAL NOTES ON OUTLINE: The first two chapters/sections of the course focus on fundamentals. Convex analysis provides the theoretical underpinnings and terminology for the optimization of continuous problems, as well as many areas in applied mathematics. The section on numerical algorithms provides a working knowledge in at least one prominent optimization technique. Students should be able to use this technique in practice.

Updated by Professor D. Shirokoff- 1/10/2021
Department of Mathematical Sciences Course Syllabus, Spring 2021