Math 222: Diff. Eqns. Final Exam May 4, 2018

Name: (print) ______________________________

Student ID number: ______________________________

Section Number: ______________________________

Signature*: ______________________________

*My signature affirms that this examination is completed in accordance with the NJIT Academic Integrity Code.

Instructions: Please complete the problems on the following pages in the space provided. If you need additional space to work, please use the back of the previous page. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, smartphones, smartwatches, CB radios, or any other external sources of information is not permitted during this examination.

Question	Points	Score
1	10	
2	10	
3	10	
4	12	
5	10	
6	10	
7	10	
8	10	
9	10	
10	8	
Total:	100	

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}, \quad s > 0$
2.	e^{at}	$\frac{1}{s-a}, \quad s > a$
3.	$t^n, \quad n = \text{positive integer}$	$\frac{n!}{s^{n+1}}, \quad s > 0$
4.	$t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5.	$\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$
6.	$\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$
7.	$\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a\rvert$
8.	$\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a\rvert$
9.	$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
10.	$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
11.	$t^n e^{at}, \quad n = \text{positive integer}$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12.	$u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$
16.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$

1. (10 points) Consider the ordinary differential equation

$$(x^2 + 1)y''(x) - 4xy'(x) + 6y(x) = 0.$$

   Solve the system using the power series method. Usually we ask for the first few terms in the power series solution, but here we are asking for *all* the terms in the series. This problem has a surprisingly simple answer because something special happens.

2. (10 points) Solve the nonhomogeneous boundary value problem. Do not use Fourier series methods.

$$y'' - y = 3x; -\ln 2 < x < \ln 2$$
$$y(-\ln 2) = y(\ln 2) = 0$$

3. (a) (5 points) What is the definition of the Laplace transform $F(s) = \mathcal{L}(f(t))$ for a given function $f(t)$?

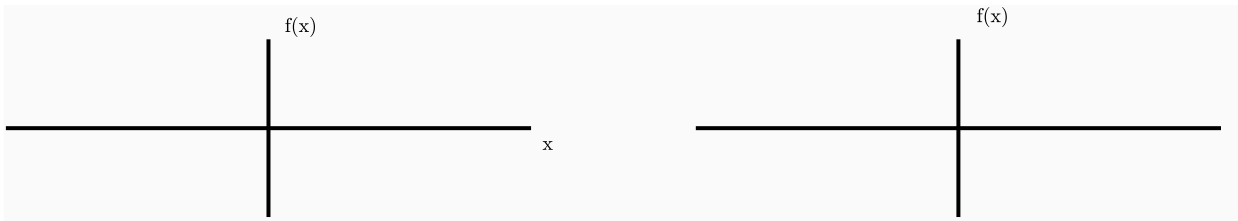
(b) (5 points) Evaluate the following definite integral:

$$\int_0^\infty e^{-(s+3)t} \cos 5t dt.$$

4. Consider the function defined on the interval $0 \leq x \leq \pi$.

$$f(x) = \begin{cases} 1; & 0 \leq x < \frac{\pi}{2}; \\ 0; & \frac{\pi}{2} \leq x \leq \pi. \end{cases}$$

(a) (4 points) On the left axis, plot three periods of the even periodic extension of $f(x)$. On the right axis do the same for the odd periodic extension.



(b) (4 points) Find the Fourier sine series of $f(x)$.

(c) (4 points) Solve the BVP for $u(x)$ using a Fourier *sine* series expansion.

$$u''(x) = f(x);\ 0 < x < \pi$$
$$u(0) = 0; u(\pi) = 0.$$

5. (10 points) Solve the following initial-value problem:

$$t(\ln t)y' + y = t, y(e) = 1.$$

6. (10 points) Solve the initial value problem:

$$y''(t) + 9y(t) = u_\pi(t)\sin(t-\pi);$$
$$y(0) = 0; y'(0) = 0.$$

7. (10 points) For what non-negative values of α does the following problem have a unique solution?

$$y''(x) + \alpha^2 y(x) = 0; 0 < x < \pi$$
$$y(0) = 0; y'(\pi) = 1.$$

Note the boundary conditions are different at the two ends!

8. (10 points) The differential equation

$$y''(t) - ty'(t) + y(t) = 0$$

has one solution $y_1(t) = t$. Find another. You should leave your answer in the form of an integral.

9. Consider the differential equation

$$\frac{d}{dt}\vec{x} = \begin{pmatrix} 2 & 2 \\ 2 & 5 \end{pmatrix} \vec{x}.$$

(a) (5 points) Find two solutions and show they are linearly independent.

(b) (5 points) Find the solution with initial condition $\vec{x}(0) = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$.

10. (8 points) Consider the differential equation

$$\frac{d}{dt}\vec{x} = A\vec{x}.$$

Below are four matrices A and six different descriptions of the behavior of its solutions. Each description matches at most one matrix. Note there are 6 choices but only 4 right answers. Please place your answer in the space provided. No partial credit.

(a) __________ $A = \begin{pmatrix} -5 & -2 \\ -2 & -2 \end{pmatrix}.$

(b) __________ $A = \begin{pmatrix} 1 & 2 \\ -5 & 3 \end{pmatrix}.$

(c) __________ $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}.$

(d) __________ $A = \begin{pmatrix} -1 & 2 \\ -5 & 1 \end{pmatrix}.$

(i) Every solution approaches 0 as $t \to \infty$, with no oscillations.

(ii) Has a nonzero solution that approaches 0 as $t \to \infty$ and has a nonzero solution that approaches ∞ as $t \to \infty$.

(iii) Every nonzero solution diverges as $t \to \infty$.

(iv) Every nonzero solution has oscillations which become progressively larger as $t \to \infty$.

(v) Every nonzero solution has oscillations which become progressively smaller as $t \to \infty$.

(vi) Every nonzero solution oscillates with constant amplitude.