Math 222: Diff. Eqns. Final Exam December 15, 2017

Name: (print) ______________________________

Student ID number: ______________________________

Section Number: ______________________________

Signature*: ______________________________

*My signature affirms that this examination is completed in accordance with the NJIT Academic Integrity Code.

Instructions: Please complete the problems on the following pages in the space provided. If you need additional space to work, please use the back of the previous page. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, smartphones, smartwatches, CB radios, or any other external sources of information is not permitted during this examination.

Question	Points	Score
1	10	
2	10	
3	8	
4	12	
5	12	
6	10	
7	8	
8	15	
9	15	
Total:	100	

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}, \quad s > 0$
2.	e^{at}	$\dfrac{1}{s-a}, \quad s > a$
3.	$t^n, \quad n = \text{positive integer}$	$\dfrac{n!}{s^{n+1}}, \quad s > 0$
4.	$t^p, \quad p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}, \quad s > 0$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}, \quad s > 0$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}, \quad s > \lvert a\rvert$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}, \quad s > \lvert a\rvert$
9.	$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}, \quad s > a$
10.	$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}, \quad s > a$
11.	$t^n e^{at}, \quad n = \text{positive integer}$	$\dfrac{n!}{(s-a)^{n+1}}, \quad s > a$
12.	$u_c(t)$	$\dfrac{e^{-cs}}{s}, \quad s > 0$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \quad c > 0$
16.	$\displaystyle\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$

1. (a) (5 points) Find the general solution to $y'' + 5y' + 4y = t^2 + 1$.

   (b) (5 points) Find a suitable integrating factor that can be used to solve the equation

$$t^2 y' - 4ty = t^2 \cos 3t.$$

2. (10 points) Evaluate the following definite integral:

$$\int_0^\infty e^{-(s+3)t} \cos 5t dt.$$

3. (8 points) Consider the autonomous equation

$$y' = y^2(y-2)(y-3).$$

Suppose $y(t)$ is a solution and $y(3) = 1$, then what is the limit of $y(t)$ as $t \to \infty$? (Hint: don't even think about finding the solution!)

4. (12 points) Solve the differential equation

$$y'' + 2xy' + xy = 0$$

using the method of infinite series. Your answer should include the expansion up to and including x^4 as well as the recurrence relation to generate the coefficients.

5. (a) (6 points) Find the inverse Laplace transform of

$$F(s) = \frac{e^{-2s}(3s+2)}{s^2+16}.$$

(b) (6 points) Solve the following system using a convolution integral

$$9y'' + 6y' + y = h(t);\; y(0) = y'(0) = 0.$$

6. (10 points) Consider all the nonzero solutions of the linear system

$$\mathbf{x}' = \begin{pmatrix} 1 & 3 \\ 3 & -1 \end{pmatrix} \mathbf{x}.$$

Which (one) of the following is true as $t \to \infty$?

(a) Some converge to $(0, 0)$, the others move away, unbounded, from $(0, 0)$.

(b) They all converge to $(0, 0)$.

(c) They all move away, unbounded, from $(0, 0)$.

(d) they neither converge to $(0, 0)$, nor move away unbounded from $(0, 0)$.

7. (8 points) Write the following as a system of first-order equations

$$(t+1)^2\frac{d^3y}{dt^3}+\frac{d^2y}{dt^2}+2\frac{dy}{dt}+6y(t)=\sin t.$$

8. Consider the equation

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 1 & -1 \\ a & -3 \end{pmatrix} \mathbf{x}.$$

(a) (5 points) For what values of the constant a does the above equation have solutions exhibiting exponentially damped oscillations?

(b) (10 points) Solve the system when $a = 8$ and the initial conditions are $(x(0), y(0)) = (2, 0)$.

9. Let $f(x) = 4 - x^2$, $0 < x < 2$.
   (a) (3 points) Consider the odd periodic extension, of period $T = 4$, of $f(x)$. Sketch 3 periods, on the interval $-6 < x < 6$, of this function.

   (b) (3 points) To what value does the Fourier series of this odd periodic extension converge at $x = -1$? At $x = 4$?

   (c) (3 points) Consider the even periodic extension, of period $T = 4$, of $f(x)$. Sketch 3 periods, on the interval $-6 < x < 6$, of this function.

   (d) (3 points) Find a_0, the constant term of the Fourier series of the even periodic extension of $f(x)$.

   (e) (3 points) For the even periodic function in part (c), write down the integral that defines a_n, the nth Fourier cosine coefficient, but do not attempt to evaluate it.