NJIT Math 222 Exam 3 April 18, 2018

Name: (print) ______________________________

Student ID number: ______________________________

Section Number: ______________________________

Signature*: ______________________________

*My signature affirms that this examination is completed in accordance with the NJIT Academic Integrity Code.

Instructions: Please complete the problems on the following pages in the space provided. If you need additional space to work, please use the back of the previous page. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, smartphones, smartwatches, or any other external sources of information is not permitted during this examination. *On your desk you may have only the exam, writing implements, and erasers.* You have 85 minutes for this test.

Question	Points	Score
1	15	
2	12	
3	16	
4	16	
5	10	
6	10	
7	10	
8	12	
Total:	101	

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}, \quad s > 0$
2.	e^{at}	$\frac{1}{s-a}, \quad s > a$
3.	$t^n, \quad n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$
4.	$t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5.	$\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$
6.	$\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$
7.	$\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a\rvert$
8.	$\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a\rvert$
9.	$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
10.	$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
11.	$t^n e^{at}, \quad n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12.	$u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$
16.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$

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1. Consider the differential equation

$$y''(x) + xy'(x) + 2y(x) = 0.$$

(a) (5 points) Find the recursion relation satisfied by the coefficients a_j for the power series solution about $x = 0$.

(b) (6 points) Solve this recursion relation up to the x^6 term.

(c) (4 points) If $y(0) = 1$ and $y'(0) = 6$, calculate $y'''(0)$.

2. (12 points) Use the Laplace transform to solve the initial value problem

$$y'' + 5y' - 6y = \delta(t-3),\ y(0) = 0,\ y'(0) = 0.$$

No credit will be given if any other method is used.

3. Consider the function defined piecewise by

$$f(t) = \begin{cases} 0 & \text{if } 0 \le t < 1; \\ e^{-t} & \text{if } 1 \le t < 2; \\ 0 & \text{if } 2 \le t. \end{cases}$$

(a) (4 points) Plot $f(t)$.

(b) (6 points) Rewrite the function $f(t)$ using step functions, thereby eliminating the need for the piecewise method of definition used above.

(c) (6 points) Find $F(s) = \mathcal{L}(f(t))$.

4. Find the inverse Laplace transforms of the following functions

   (a) (8 points) $F(s) = e^{-4s}\dfrac{s+6}{s^2+2s+26}$.

   (b) (8 points) $F(s) = \dfrac{s^2-2s-1}{s(s^2+1)}$.

5. (10 points) If $f(t) = t$ and $g(t) = \frac{1}{t+1}$, what is their convolution $(f * g)(t)$? Set up, but do not evaluate any integrals that appear in your answer.

6. (10 points) Solve the initial value problem

$$y''(t) + 9y(t) = \sin^{2018} t;$$
$$y(0) = y'(0) = 0.$$

Hint: Your answer should be in the form of a single definite integral.

7. (10 points) Rewrite following the initial value problem as a first-order system of differential equations and initial values:

$$y'''(x) + \sin x \cdot y'(x) + 5y(x) = 10;$$
$$y(0) = 0, \; y'(0) = 1, \; y''(0) = 2.$$

8. (12 points) Let

$$A = \begin{pmatrix} 3 & 4 & 5 \\ 3 & 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 \\ -1 & 2 \end{pmatrix}, C = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}, \vec{x} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}.$$

Compute the following if they are well-defined or else state that they are undefined (and why).

(a) AB

(b) BA

(c) $B + A$

(d) $B + C$

(e) $A\vec{x}$

(f) $B\vec{x}$