Math 222: Diff. Eqns. Common Exam 3 November 20, 2019

Name: (print) ______________________________

Student ID number: ______________________________

Section Number: ______________________________

Signature*: ______________________________

*My signature affirms that this examination is completed in accordance with the NJIT Academic Integrity Code.

Instructions:

Please complete the problems on the following pages in the space provided. If you need additional space to work, please use the back of the previous page. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, or any other external sources of information is not permitted during this examination.

The exam writer's promise: All the algebra works out neatly. If you ever find yourself with a "funny" number like $\sqrt{31}$ or $\frac{13}{47}$, you have made an algebra error.

Question	Points	Score
1	20	
2	15	
3	20	
4	20	
5	10	
6	15	
Total:	100	

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}, \quad s > 0$
2. e^{at}	$\frac{1}{s-a}, \quad s > a$
3. $t^n, \quad n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$
4. $t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5. $\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$
6. $\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$
7. $\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a\rvert$
8. $\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a\rvert$
9. $e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
10. $e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
11. $t^n e^{at}, \quad n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$
14. $e^{ct}f(t)$	$F(s-c)$
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$
16. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$

1. (20 points) Consider the differential equation

$$\left(x^2+1\right)y''+6xy'+4y=0$$

with initial conditions

$$y(0)=1,\ y'(0)=0.$$

Show that the first five nonzero terms in the power series solution for this problem agree with the exact solution

$$y=\sum_{k=0}^{\infty}(-1)^k(k+1)x^{2k}.$$

2. (a) (10 points) **Using the definition of the Laplace transform,** calculate the Laplace transform $\mathcal{L}\{f(t)\} = F(s)$ of

$$f(t) = e^{12t}.$$

Obviously, the answer is in the table, but zero credit will be given without a proper computation.

(b) (5 points) For what values of s is $F(s)$ well-defined?

3. (a) (10 points) Determine the following Laplace transform. You may use the table, but you should state clearly your reasoning at each step of the calculation and which, if any, items from the table you are using.

$$\mathcal{L}\left\{e^{-10t}u_3(t)\,(t-3)^3\right\}$$

(b) (10 points) Compute $f(2\pi)$, given that

$$f(t) = 2 + t + u_3(t)\cos t + u_{10}(t)t^2.$$

4. (20 points) Use the Laplace transform to solve the following initial value problem:

$$y'' + 9y = 6\ \delta(t-7),$$
$$y(0) = 0,\ y'(0) = 4.$$

5. (10 points) Compute $\mathcal{L}^{-1}\{F(s)\}$ if

$$F(s) = \frac{10s - 20}{s^2 - 25}.$$

6. (a) (10 points) Express the solution of the following IVP as the convolution of two functions

$$y''(t) + 6y'(t) + 10y(t) = e^{-t^2},$$
$$y(0) = y'(0) = 0.$$

Note that *you do not know the Laplace transform of the right hand side*, so that while direct solution by Laplace transform is not an option, convolution at least gives you some sort of formula.

(b) (5 points) Write down the integral defining this convolution but do not attempt to integrate it.