Math 222: Diff. Eqns. Common Exam 3 November 29, 2017

Name: (print) ______________________________

Student ID number: ______________________________

Section Number: ______________________________

Signature*: ______________________________

*My signature affirms that this examination is completed in accordance with the NJIT Academic Integrity Code.

Instructions:

Please complete the problems on the following pages in the space provided. If you need additional space to work, please use the back of the previous page. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, or any other external sources of information is not permitted during this examination.

Question	Points	Score
1	20	
2	15	
3	20	
4	15	
5	15	
6	15	
Total:	100	

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}, \quad s > 0$
2.	e^{at}	$\frac{1}{s-a}, \quad s > a$
3.	$t^n, \quad n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$
4.	$t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5.	$\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$
6.	$\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$
7.	$\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a \rvert$
8.	$\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a \rvert$
9.	$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
10.	$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
11.	$t^n e^{at}, \quad n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12.	$u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$
16.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$

1. (20 points) Construct a power series solution, including terms up to x^4 for the initial value problem

$$y''(x) + y'(x) + 2xy(x) = x + 1;$$
$$y(0) = 3; y'(0) = 1.$$

2. (15 points) Solve the initial-value problem for the Euler differential equation:

$$x^2y''(x) + 3xy'(x) + 2y(x) = 0, y(1) = 1,\ y'(1) = 1,$$

and describe the behavior of solutions as $x \to 0^+$.

3. (a) (10 points) Determine the following Laplace transform:

$$\mathcal{L}\left\{e^{-10t}\,(t+5)^2\right\}$$

(b) (10 points) Compute $f(4)$, given that

$$f(t) = 2 + u_3(t)(t-1) + u_5(t)t^2.$$

4. (15 points) Use the Laplace transform to solve the following initial value problem.

$$y'' + 4y = 2\delta(t-5) - 8u_3(t),$$
$$y(0) = 0, y'(0) = 4.$$

5. (15 points) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix}.$$

6. (a) (7 points) For what value(s) of a are the vectors $\mathbf{u} = (4,5)$ and $\mathbf{v} = (-1,a)$ linearly dependent?

(b) (8 points) Solve the linear system

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 6 \\ 2 \\ 0 \end{pmatrix}.$$