NJIT Math 222 Exam 2 March 7, 2018

Name: (print) _____

Student ID number: _____

Section Number: _____

Signature*: _____

*My signature affirms that this examination is completed in accordance with the NJIT Academic Integrity Code.

Instructions: Please complete the problems on the following pages in the space provided. If you need additional space to work, please use the back of the previous page. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, smartphones, smartwatches, or any other external sources of information is not permitted during this examination. *On your desk you may have only the exam, writing implements, and erasers.* You have 85 minutes for this test.

Question	Points	Score
1	10	
2	15	
3	15	
4	19	
5	15	
6	15	
7	15	
Total:	104	

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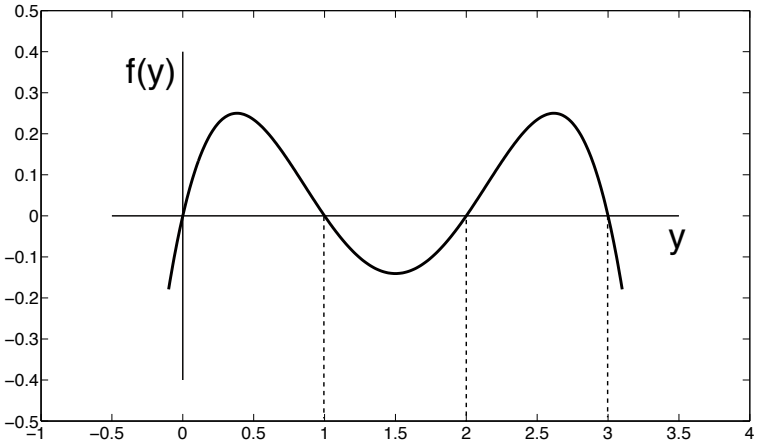

1. (10 points) Consider the autonomous ODE

$$\frac{dy}{dt} = f(y),$$

where $f(y) = -y(y-1)(y-2)(y-3)/4$. A sketch of $f(y)$ is given above. Place a check mark in front of all the true statements below:

- ☐ The ODE has exactly two stable equilibrium solutions.
- ☐ The ODE has exactly four unstable equilibrium solutions.
- ☐ All solutions of this ODE approach $-\infty$, as $t \to \infty$.
- ☐ All negative solutions of this ODE approach $-\infty$, as $t \to +\infty$.
- ☐ If $y(1) = 1.5$, then there exists a time t_0 such that $y(t_0) = 2.5$.

2. (15 points) Consider the initial value problem

$$\frac{dy}{dt} = t - y^2 + 1,\ y(0) = 1.$$

Use Eulers method with step size $h = 0.1$ to find the approximate value of the solution of the initial value problem up to time $t = 0.2$.

3. A tank initially contains 100 liters of fresh water. A mixture containing 10 grams of salt per liter is poured into the tank at the rate of 2 liters per minute. The well-stirred mixture is allowed to leave the tank at the same rate.

   (a) (2 points) Introduce the variables, give their units, and their meaning.

   (b) (4 points) Write the differential equation satisfied by the mass in the tank, and give the initial conditions, that describe this event.

   (c) (9 points) Solve the initial value problem.

4. (4 points) (a) (7 points) Find the general solution to

$$y''(t) + 12y'(t) + 36y(t) = 0.$$

(b) (4 points) Write down an appropriate guess for the particular solution $Y(t)$ for the following equation using the method of undetermined coefficients:

$$y''(t) + 3y'(t) - 4y(t) = e^{4t}t^2.$$

(c) (4 points) Formulate a guess for the particular solution $Y(t)$ for the following equation using the method of undetermined coefficients:

$$y''(t) + 3y'(t) - 4y(t) = e^{-4t}t^2.$$

5. (15 points) Given that $y_1(t) = (t+4)^3$ is a known solution of the linear differential equation

$$(t+4)^2 y''(t) - 5(t+4)y'(t) + 9y(t) = 0, t > -4,$$

find the general solution of the equation.

6. (15 points) Find the general solution to

$$x''(t) + 4x'(t) + 4x(t) = \frac{e^{-2t}}{t}, t > 0.$$

7. A mass that weighs 8 kg stretches a frictionless spring $\frac{5}{8}$ m under gravity. The system is acted on by an external force $F = 72 \sin 5t$. At $t = 0$, the mass is pulled down three inches from its equilibrium and released. Use the approximation that gravitational acceleration is $g = 10$ m/s^2.

   (a) (2 points) What is the spring constant k? Be careful about units.

   (b) (3 points) Write down the initial value ODE problem solved by the displacement $u(t)$.

   (c) (8 points) Solve.

   (d) (2 points) What is the beat frequency of the solution?