Math 222: Diff. Eqns. Common Exam 2 October 23, 2019

Name: (print) ______________________________

Student ID number: ______________________________

Section Number: ______________________________

Signature*: ______________________________

*My signature affirms that this examination is completed in accordance with the NJIT Academic Integrity Code.

Instructions:

Please complete the problems on the following pages in the space provided. If you need additional space to work, please use the back of the previous page. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, or any other external sources of information is not permitted during this examination.

Question	Points	Score
1	15	
2	15	
3	12	
4	18	
5	20	
6	20	
Total:	100	

1. (15 points) Match the differential equations listed below with the descriptions of long time behavior listed below. (Each description (i through vi) matches at most one equation. Please place your answer in the space provided. No partial credit)

(a) ____________ $y'' - 6y' + 25y = 0.$

(b) ____________ $3y'' + 12y = 0.$

(c) ____________ $y'' - 3y' - 4y = 0.$

(d) ____________ $y'' + 10y' + 25y = 0.$

(i) Every solution approaches 0 as $t \to \infty$.

(ii) Has a nonzero solution that approaches 0 as $t \to \infty$ and also has a nonzero solution that approaches ∞ as $t \to \infty$.

(iii) Every nonzero solution approaches either ∞ or $-\infty$ as $t \to \infty$.

(iv) Every nonzero solution has oscillations which become progressively larger as $t \to \infty$.

(v) Every nonzero solution has oscillations which become progressively smaller as $t \to \infty$.

(vi) Every nonzero solution oscillates with constant amplitude as $t \to \infty$.

2. (15 points) The ODE

$$t^2y'' - 5ty' + 8y = 0, t > 0$$

has one solution $y_1 = t^2$. Use the method of reduction of order to find another solution y_2 of this linear homogeneous ODE that is not a constant multiple of y_1.

3. (12 points) A mass weighing 16 lb stretches a spring 8 in. There is no damping. At the initial time $t = 0$ the mass is pushed upward, contracting the spring by a length of 6 in, and set in motion with a downward velocity of 2 ft/s. Find the values of the mass and spring constant and set up the initial value problem but **do not solve**. You may assume that gravitational acceleration is 32 ft/s^2.

4. A spring-mass system is modeled by the initial value problem

$$3y'' + \gamma y' + 75y = F(t), \gamma \geq 0, y(0) = 3, y'(0) = -15.$$

(a) (5 points) If $\gamma = 0$ and $F(t) = 0$, solve the initial value problem.

(b) (3 points) Put the answer from part (a) in amplitude-phase form.

(c) (5 points) If $\gamma = 0$ and $F(t) = 3\cos(\omega t)$, for which value(s) of $\omega > 0$ will the system undergo resonance?

(d) (5 points) If $F(t) = 0$, for which value(s) of γ will the system be critically damped?

5. Consider the differential equation

$$y''(t) + 4y'[t] + 3y(t) = g(t).$$

(a) (8 points) Find two solutions to the associated homogeneous equation, and demonstrate they are a fundamental solution set.

(b) (12 points) Solve the given system when $g(t) = (-2+8t)e^t$ and the initial conditions are $y(0) = 0; y'(0) = 0$.

6. (20 points) Find the general solution to the differential equation

$$y''(t) + 4y'(t) + 4y(t) = \frac{e^{-2t}}{t^2+1},$$

given that the associated homogeneous equation has a fundamental solution pair

$$y_1(t) = e^{-2t}, y_2(t) = te^{-2t}.$$