Name: (print) ______________________________

Student ID number: ______________________________

Section Number: ______________________________

Signature*: ______________________________

*My signature affirms that this examination is completed in accordance with the NJIT Academic Integrity Code.

Instructions:

Please complete the problems on the following pages in the space provided. If you need additional space to work, please use the back of the previous page. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, or any other external sources of information is not permitted during this examination.

Question	Points	Score
1	12	
2	15	
3	15	
4	18	
5	20	
6	20	
Total:	100	

1. (12 points) Match the differential equations listed below with the descriptions of long time behavior listed below. (Each description matches only one equation. Please place your answer in the space provided. No partial credit)

   (a) ____________ $y'' - y' - 2y = 0.$

   (b) ____________ $y'' + 4y' + 4y = 0.$

   (c) ____________ $y'' - 4y' + 29y = 0.$

   (d) ____________ $y'' + y = 0.$

   (i) Every solution approaches 0 as $t \to \infty$.

   (ii) Has a nonzero solution that approaches 0 as $t \to \infty$ and has a nonzero solution that approaches ∞ as $t \to \infty$.

   (iii) Every nonzero solution approaches either ∞ or $-\infty$ as $t \to \infty$.

   (iv) Every nonzero solution has oscillations which become progressively larger as $t \to \infty$.

   (v) Every nonzero solution has oscillations which become progressively smaller as $t \to \infty$.

   (vi) Every nonzero solution oscillates with constant amplitude as $t \to \infty$.

2. (15 points) The ODE
$$t^2y'' - ty' + y = 0, t > 0$$
obviously has a solution $y_1 = t$. Use the method of reduction of order to find another solution of this linear homogeneous ODE that is not a constant multiple of y_1.

3. (15 points) What is the largest interval on which the following initial value problem is guaranteed to have a solution?
$$(t^2 - 4)y' + 2ty = \frac{1}{t^2}; \quad y(1) = 4.$$

4. (18 points) A spring-mass system is modeled by the initial value problem
$$2y'' + \gamma y' + 8y = F(t), \gamma \geq 0, y(0) = 3, y'(0) = -4.$$

   (a) If $\gamma = 0$ and $F(t) = 0$, what is the amplitude of displacement?

   (b) If $\gamma = 0$ and $F(t) = 3\cos(\omega t)$, for which value(s) of ω will the system undergo resonance?

   (c) If $F(t) = 0$, for which value(s) of γ will the system be critically damped?

5. (20 points) Consider the differential equation
$$y''(t) + 4y(t) = g(t).$$

(a) Find two solutions to the associated homogeneous equation, and demonstrate they are a fundamental solution pair.

(b) Solve the given system when $g(t) = (2 + 5t)e^t$ and the initial conditions are $y(0) = 0; y'(0) = 0$.

6. (20 points) Find the general solution to the differential equation

$$y''(t) - 2y'(t) + y(t) = \frac{e^t}{t},$$

given that the associated homogeneous equation has a fundamental solution pair

$$y_1(t) = e^t, y_2(t) = te^t.$$