Name: (print) ______________________________

Student ID number: ______________________________

Section Number: ______________________________

Signature*: ______________________________

*My signature affirms that this examination is completed in accordance with the NJIT Academic Integrity Code.

Instructions: Please complete the problems on the following pages in the space provided. If you need additional space to work, please use the back of the previous page. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, smartphones, smartwatches, or any other external sources of information is not permitted during this examination. *On your desk you may have only the exam, writing implements, and erasers.* You have 85 minutes for this test. Happy Valentine's Day. ♡

Question	Points	Score
1	12	
2	10	
3	16	
4	10	
5	17	
6	15	
7	20	
Total:	100	

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1. (12 points) Classify the following equations as linear or nonlinear, and state their order.

Equation	Linear or nonlinear?	Order
$t\frac{d^2y}{dt^2} + t^2\frac{dy}{dt} + t^3y = \cos t.$		
$t\frac{d^2y}{dt^2} + t^2\frac{dy}{dt} + t^3y = \cos y.$		
$\frac{dy}{dx} = \frac{2y-2}{2x+3}.$		

2. (10 points) Find the solution to:

$$\frac{dy}{dx} = \frac{x}{2y^2};\ y(0) = 1.$$

3. (a) (10 points) Find the solution to the initial value problem

$$y'' + 3y' + 2y = 0;\ y(0) = 0;\ y'(0) = \alpha.$$

(b) (3 points) For what value(s) of α is $\lim_{t\to\infty} y(t) = 0$?

(c) (3 points) For what value(s) of α is $\lim_{t\to-\infty} y(t) = 0$?

4. Consider the autonomous equation $y' = y^2 - 3y$.
   (a) (4 points) Find the equilibrium solutions.

   (b) (4 points) Classify all equilibria as unstable or asymptotically stable.

   (c) (2 points) If $y(0) = 0$, what is $y(1)$?

5. Consider the initial value problem

$$y' - \tan(t)y = \frac{3t}{\cos(t)};\ y(0) = 4.$$

(a) (4 points) On what interval is the solution guaranteed to exist by theorems in the book? Why?

(b) (10 points) Solve the problem.

(c) (3 points) What is the maximum interval of existence for the solution you found. Does it match what you found in the first part?

6. (a) (10 points) Solve the initial value problem

$$y''(x) + 2y'(x) + 10y(x) = 0,\ y(0) = 3,\ y'(0) = 0.$$

(b) (5 points) Describe in a brief sentence the type of behavior exhibited by the solution $y(x)$.

7. (20 points) Show that $y_1(t) = t^2$ and $y_2(t) = t^3$, $t > 0$ form a fundamental set of solutions for the differential equation $t^2y'' - 4ty' + 6y = 0$.