NJIT Math 222 Exam 1 September 25, 2019

Name: (print) _____

Student ID number: _____

Section Number: _____

Signature*: _____

*My signature affirms that this examination is completed in accordance with the NJIT Academic Integrity Code.

Instructions: Please complete the problems on the following pages in the space provided. If you need additional space to work, please use the back of the previous page. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, smartphones, smartwatches, or any other external sources of information is not permitted during this examination. *On your desk you may have only the exam, writing implements, and erasers.* You have 85 minutes for this test.

Question	Points	Score
1	10	
2	10	
3	15	
4	15	
5	15	
6	15	
7	10	
Total:	90	

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1. (10 points) Classify the following equations as linear or nonlinear, and state their order.

Equation	Linear or nonlinear?	Order
$t^4\frac{d^2y}{dt^2} + t^3\frac{dy}{dt} + t^2y = \cos t.$		
$t^4\frac{d^3y}{dt^2} + t^3\frac{d^2y}{dt^2} + t^2y = \cos y.$		
$\frac{dy}{dx} = \frac{2xy-2}{2e^x+3}.$		

2. (10 points) Verify that the function $y(x) = x^2e^x$ satisfies the initial value problem:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 2e^x$$
$$y(0) = 0, \quad \frac{dy}{dx}(0) = 0$$

3. (a) (5 points) Solve the initial value problem

$$y' - \frac{3}{2}y = 3 + 2e^t,\ y(0) = y_0.$$

(b) (5 points) What the value of y_0 separates solutions that grow positively as $t \to \infty$ from those that grow negatively.

(c) (5 points) How does the solution that corresponds to this critical value of y_0 behave as $t \to \infty$?

4. Consider the differential equation

$$y' = (3 + y)\left(4 - y^2\right)$$

(a) (5 points) Draw a direction field in the space below. Clearly label your axes and make sure you choose a large enough range of y-values to demonstrate all the different types of behavior present in the system.

(b) (5 points) Based on the direction field, determine the behavior of y as $t \to \infty$. If this behavior depends on the value of y at $t = 0$, describe this dependency.

(c) (5 points) On the axis above, sketch the solution with the initial condition

$$y(0) = 5.$$

5. (15 points) Consider the initial value problem

$$\frac{dy}{dt} = y^2 - t - 4,\ y(0) = 2.$$

Use Euler's method with step size $h = \frac{1}{4}$ to find the approximate value of the solution of the initial value problem up to time $t = \frac{1}{2}$. Express your answer as a fraction.

6. (15 points) (a) Find the explicit form of the solution of the initial value problem

$$y' = \frac{x(x^2+1)}{4y^3},\ y(0) = \frac{-1}{\sqrt{2}}.$$

(b) On what interval is the solution is defined?

7. Some nuclei are energetically unstable and can spontaneously transform into more stable forms by something known as radioactive decay. The rate at which a particular radioactive sample will decay depends on its identity. Tables have been compiled which list the half?lives of various radioisotopes.

   The rate at which a sample decays is proportional to the amount of the sample present. Therefore, if $x(t)$ denotes the amount of a radioactive substance at time t, then

   $$\frac{dx}{dt} = -kx, \ (k > 0).$$

   (a) (2 points) What are the dimensions of the coefficient k.

   (b) (2 points) If time is measured in years, what are the units of k? (Note the distinction between this question and the one above it.

   (c) (4 points) The *half-life* of a sample is the amount of time T_{half} is the time it takes for a sample with initial quantity of an isotope $x(0) = X$ to decay to $x(T_{\text{half}}) = \frac{X}{2}$. Solve the differential equation and find T_{half} in terms of k.

   (d) (2 points) The half-life of carbon-14 is $T_{\text{half}} = 5730$ years. What is k for this isotope?