Math 222: Diff. Eqns. Common Exam 1 September 27, 2017

Name: (print) _____

Student ID number: _____

Section Number: _____

Signature*: _____

*My signature affirms that this examination is completed in accordance with the NJIT Academic Integrity Code.

Instructions:

Please complete the problems on the following pages in the space provided. If you need additional space to work, please use the back of the previous page. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, or any other external sources of information is not permitted during this examination.

Question	Points	Score
1	10	
2	20	
3	15	
4	15	
5	20	
6	20	
Total:	100	

1. (10 points) What is the order of each of the following ODE. Indicate whether each is linear or nonlinear.

   (a)
   $$y''''(x) = y(x)y'(x).$$

   (b)
   $$x^3\frac{d^3y}{dx^3} + x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y(x) = e^x.$$

2. (20 points) Consider the differential equation
   $$\frac{dy}{dt} = y - y^3.$$

   (a) Identify all equilibria of this system.

(b) Draw the direction field for this problem for $-2 \leq y \leq 2$ and $t > 0$. Include a sufficient number of integral curves to give a sense of the dynamics.

(c) Describe the behavior of solutions as $t \to \infty$. In particular, describe how the behavior depends on the initial conditions.

3. (15 points) Solve the differential equation

$$y' - \frac{2}{t}y = t^2 e^t$$

subject to the condition $y(1) = 3$.

4. (a) (10 points) Solve the differential equation

$$y'' + y' - 6y = 0$$

subject to the conditions $y(0) = 3$ and $\lim_{t\to\infty} y(t) = 0$.

(b) (5 points) What is the values of $y'(0)$ in this case?

5. (20 points) Consider the differential equation

$$\frac{dy}{dt} = 6 - 4y + 3t$$

subject to the initial condition $y(0) = 3$. Use the Euler method to generate approximate a solution up to time $t = 1$ using a step size of $h = \frac{1}{3}$.

6. (20 points) Radiocarbon dating, used in archeology and anthropology, is based on the radioactive decay of carbon-14 which appears in trace amounts in all living tissue. If $Q(t)$ is the amount of carbon-14 in a sample, then $Q(t)$ evolves according to

$$\frac{dQ}{dt} = -rQ.$$

(a) Suppose you are trying to date a scrap of papyrus found in a sample buried in an ancient Egyptian pyramid. State the dimensions and some appropriate units for the quantities Q, r, and t.

(b) If the quantity of carbon-14 at $t = 0$ is Q_0, what is $Q(t)$?

(c) The half life is the time t_{half} for the amount of a radioactive material to decay by half, i.e. such that $Q(t_{\text{half}}) = Q_0/2$. If $t_{\text{half}} = 5730$ years. What is r? Be sure to include units in your answer that match part (a).