Math 213 Final Exam
December 16, 2019

Vector quantities are in boldface. Calculators are NOT allowed. 11 problems in total.

Multiple Choice with work needing to be shown

For each of the following multiple choice problems, clearly indicate your answer by writing your solution choice, A, B, C or D in capital letters in your exam booklet and placing a box or circle around it. No credit will be given unless your work justifies the answer. No partial credit will be given, so make sure you carefully check your work. Each problem is worth 10 points.

1. Find the directional derivative of the function $f(x, y, z) = xy + yz + zx$ at $P_0 = (1, -1, 2)$ in the direction of $\mathbf{v} = 3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$

   (A) 1
   (B) 2
   (C) 3
   (D) 4

2. Find the equation of the plane containing the three points A=(5,2,1), B=(3,-3,2) and C=(0,4,2).

   (A) $7x - 7y - 21z = 0$
   (B) $7x - 7y - 21z = -70$
   (C) $5x + 2y + z = 124$
   (D) $7x + 3y + 29z = 70$

3. A glider is soaring up along the helix $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$. The length of the glider's path from $t = 0$ to $t = 2\pi$ is

   (A) $\sqrt{2}$
   (B) 2π
   (C) 2
   (D) $2\pi\sqrt{2}$

4. Assume that z is a function of the two independent variables x and y. Use implicit differentiation on the equation $yz - \ln z = x^2 + y$ to show that $\partial z/\partial x$ is

   (A) $x^2 - y$
   (B) $2xz/(yz - 1)$
   (C) $-2xz$
   (D) $z/(yz - 1)$

5. Let R be the region contained within the semi-circle of radius 1 for $y \geq 0$ given by the equation $x^2 + y^2 = 1$. Use polar coordinates to show that $\iint_R e^{x^2+y^2}\, dA$ is given by

   (A) 1
   (B) $e^2 - 1$
   (C) $\pi(e - 1)/2$
   (D) $\pi/2$

6. Find the surface area of the portion of the hyperbolic paraboloid $z = y^2 - x^2$ that lies inside the cylinder $x^2 + y^2 = 2$.

   (A) $13\pi/3$
   (B) 2π
   (C) $5\pi/2$
   (D) $7\pi/2$

7. Find the value of $\int\int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\ d\sigma$ using Stokes Theorem where S is the elliptical shell $4x^2 + 9y^2 + 36z^2 = 36$, $z \geq 0$, $\mathbf{n}$ is the outward unit mormal and the vector field $F = y\mathbf{i} + x^2\mathbf{j} + (x^2 + y^4)^{3/2} \sin(e^{\sqrt{xyz}})\mathbf{k}$ (Hint: One parametrization of the ellipse at the base of the shell is $x = 3\cos t$, $y = 2\sin t$, $0 \leq t \leq 2\pi$.)

   (A) -4π
   (B) 4π
   (C) -6π
   (D) 6π

Short Answer: Please show all work. Each problem is worh 20 points.

8. Consider the triple integral given below.

$$\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} dz\ dy\ dx$$

   Rewrite the integral as an equivalent integral in the order

   (a) $dy\ dz\ dx$
   (b) $dx\ dz\ dy$

9. Let C_1 be the line segment connecting $(0,0,0)$ to $(1,1,0)$ and C_2 be the line segment connecting $(1,1,0)$ to $(1,1,1)$.

   (a) Parametrize both C_1 and C_2 and find $|\mathbf{v}|$ for each.
   (b) Show that $\int_{C_1 \cup C_2} x - 3y^2 + z\ ds = -(\sqrt{2} + 3)/2$.

10. Use a parametrization to find the outward flux $\int\int_S \mathbf{F} \cdot \mathbf{n}\ d\sigma$ of the vector field $\mathbf{F} = z^2\mathbf{i} + x\mathbf{j} - 3z\mathbf{k}$ through the surface cut from the parabolic cylinder $z = 4 - y^2$ by the planes $x = 0$, $x = 1$, and $z = 0$.

11. Use the Divergence Theorem to compute the outward flux $\int\int_S \mathbf{F} \cdot \mathbf{n}\ d\sigma$ of $\mathbf{F} = y^2z\mathbf{i} + 4yz^2\mathbf{j} + z^3\mathbf{k}$ where S is the surface of the solid sphere D given by $x^2 + y^2 + z^2 \leq 1$.