Math 213 Common Exam 3
November 20, 2019

Vector quantities are in boldface. Calculators are NOT allowed. Questions on both sides!

Multiple Choice with work needing to be shown

For each of the following multiple choice problems, clearly indicate your answer by writing your solution choice, A, B, C or D in capital letters in your exam booklet and placing a box or circle around it. No credit will be given unless your work justifies the answer. No partial credit will be given, so make sure you carefully check your work. Each problem is worth 10 points.

1. The volume of the solid bounded back and front by $x = 1$ and $x = 2$, above by $z = x + 1$, below by $z = 0$ and on the sides by the surfaces $y = 1/x$ and $y = -1/x$ is:

   (A) 1
   (B) $1 + \ln 2$
   (C) $2 - \ln 2$
   (D) $2 + 2\ln 2$

2. The value of the integral is:

$$\int_0^2 \int_0^{4-x^2} \frac{2xe^{2y}}{4-y}\, dy\, dx$$

   (A) 2
   (B) $e^2 - 1$
   (C) $(e^8 - 1)/2$
   (D) $2e^4 + 2$

3. The work done by $\mathbf{F} = xy\mathbf{i} + y\mathbf{j} - yz\mathbf{k}$ in moving a particle over the curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$, $0 \le t \le 1$ in the direction of increasing t is:

   (A) 0
   (B) 1/2
   (C) 1
   (D) 3/2

4. Convert the integral $\int_{-1}^{1} \int_0^{\sqrt{1-y^2}} \int_0^x x^2 + y^2 \; dz dx dy$ to <u>cylindrical coordinates</u> and show that its value is:

   (A) 1/5
   (B) 2/5
   (C) 3/5
   (D) 4/5

5. Using Green's Theorem, the outward flux for the field $\mathbf{F} = 2e^{xy}\mathbf{i} + y^3\mathbf{j}$ over the square bounded by the lines $x = \pm 1$, $y = \pm 1$ is:

   (A) 0
   (B) 1
   (C) 3
   (D) 4

TURN OVER

Short Answer: Please show all work.

6. Consider the integral

$$\int_0^{2/3} \int_y^{2-2y} (x + 2y)e^{y-x} dx\ dy.$$

   (a) (5 pts) Graph the domain of integration in the xy-plane.

   (b) (5 pts) Let $u = x+2y$, $v = y-x$ and graph the transformed domain G in the uv-plane.

   (c) (10 pts) Using the change of variables formula, evaluate the integral over G in the uv-plane.

7. Let $\mathbf{F} = (y^2 + 2xz)\mathbf{i} + (2xy + z)\mathbf{j} + (x^2 + y + 3z^2)\mathbf{k}$.

   (a) (5 pts) Show that $\mathbf{F}$ is a conservative vector field on R^3.

   (b) (10 pts) Find the potential function f for $\mathbf{F}$ on R^3.

   (c) (5 pts) Find the work done in moving a particle from $(0, 0, 0)$ to $(1, 0, 1)$ along the helix $\mathbf{r}(t) = \cos 2\pi t\mathbf{i} + \sin 2\pi t\mathbf{j} + t\mathbf{k}$.

8. Let D be the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq 2$.

   (a) (5 pts) Use spherical coordinates to describe the region D.

   (b) (5 pts) Evaluate $\int\int\int_D f(x, y, z)\ dV$, where $f(x, y, z) = z$.