Math 213 Common Exam 2
October 23, 2019

Vector quantities are in boldface. Calculators are NOT allowed. Questions on both sides!

Multiple Choice with work needing to be shown

For each of the following multiple choice problems, clearly indicate your answer by writing your solution choice, A, B, C or D in capital letters in your exam booklet and placing a box or circle around it. No credit will be given unless your work justifies the answer. No partial credit will be given, so make sure you carefully check your work. Each problem is worth 10 points.

1. If $f(x,y) = \frac{\sin(xy)}{x}$, assuming that $(x,y) \neq (0,0)$ then the mixed partial derivative $f_{xy} = f_{yx}$ and is

   (A) $-y\sin(xy)$
   (B) $x\cos(xy) + \sin(xy)$
   (C) $(\cos(xy))/x^2$
   (D) 0

2. The limit as $(x,y) \to (0,0)$ of the function $f(x,y) = \frac{x^4}{x^4+y^2}$ is:

   (A) 0
   (B) 1
   (C) ∞
   (D) Does Not Exist

3. The directional derivative of the function $f(x,y,z) = xy + yz + xz$ at the point $(2,-1,2)$ in the direction of $\mathbf{v} = 3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$ is:

   (A) 23/7
   (B) 25/7
   (C) 27/7
   (D) 29/7

4. Evaluate $\int_0^2 \int_{y^2}^4 y\cos\left(x^2\right)\, dxdy$ by reversing the order of integration.

   (A) $\sin 16 + 1/2\cos 16$
   (B) $4\ \sin 4$
   (C) $8\ \sin 4 - 2\cos 4$
   (D) $1/4\ \sin 16$

5. If $w = x^2 + 3xy^2$ where $x = 3r + 4s^2$ and $y = (r-s)^5$, then $\partial w/\partial r$ evaluated at $r = 1$, $s = 0$ is

   (A) 5
   (B) -7
   (C) 117
   (D) 82

TURN OVER

Short Answer: Please show all work.

6. Consider the function $f(x, y) = x^3 - 2xy + y^2 + 5$.

   (a) (10 pts) Find the equation for the tangent plane to the graph of $z = f(x, y)$ at the point $(2, 3, f(2, 3))$.

   (b) (5 pts) Calculate an estimate for the value $f(2.1, 2.9)$ using the standard linear approximation of f at $(2, 3)$.

   (c) (5 pts) Find the normal line to the zero level surface of $F(x, y, z) = f(x, y) - z$ at the point $(2, 3, f(2, 3))$

7. (15 pts) The function $f(x, y) = 4xy - x^4 - y^4$ has three critical points. Find them and use the 2nd Derivative Test to classify each as a local maximum, local minimum or saddle point.

8. (15 pts) The function $f(x, y) = e^{xy}$ subject to the constraint $x^3 + y^3 = 16$ has a local extrema with value of e^4.

   (a) Use the method of Lagrange multipliers to find the value(s) of x and y where this occurs.

   (b) By comparing to values of $f(x, y)$ at other points that satisfy the constraint, determine if this is a local maxima or local minima.

   (c) **BONUS: (5 pts)** Compute $\lim_{x\to\infty} f(x, y)$ subject to the constraint.