Math 213 Common Exam 1
September 25, 2019

Vector quantities are depicted in boldface. Calculators are NOT allowed.

Multiple Choice with work needing to be shown

For each of the following multiple choice problems, clearly indicate your answer by writing your solution choice, A, B, C or D in capital letters in your exam booklet and placing a box or circle around it. No credit will be given unless your work justifies the answer. No partial credit will be given, so make sure you carefully check your work. Each problem is worth 10 points.

1. The vector projection $\text{proj}_{\mathbf{v}}\mathbf{u}$ of $\mathbf{u} = 3\mathbf{j} + 4\mathbf{k}$ onto $\mathbf{v} = 10\mathbf{i} + 11\mathbf{j} - 2\mathbf{k}$ is

   (A) $\frac{1}{9}(10\mathbf{i} + 11\mathbf{j} - 2\mathbf{k})$
   (B) $-16\mathbf{i} + 7\mathbf{j} + \mathbf{k}$
   (C) $\frac{2}{9}(5\mathbf{i} + 11\mathbf{j} - \mathbf{k})$
   (D) None of the above

2. The parametric equations for the line that passes through P(-3,1,2) and Q(2,4,-2) are:

   (A) $x = -3 + 5t,\ y = 1 + 3t,\ z = 2 - 4t$
   (B) $x = -3 + 4t,\ y = 1 - 3t,\ z = 2 - 3t$
   (C) $x = -3 + 2t,\ y = 1 + 4t,\ z = 2 - 2t$
   (D) None of the above

3. $\mathbf{r}(t) = \tan(t)\mathbf{i} + \sec(t)\mathbf{j}$, $0 \leq t \leq \pi/4$ represents the position of a particle in the xy-plane at time t. Find the equation in x and y whose graph is the path of the particle.

   (A) $y = 1 + x^2$
   (B) $y = 1 - x^2$
   (C) $y = 1 + x$
   (D) $y = \sqrt{1 + x^2}$

4. The quadric surface given by the equation $x^2 + y^2 + 4z^2 - 2x + 4y + 1 = 0$ is:

   (A) an elliptical paraboloid with local minimum at $(1, -2, 0)$
   (B) an elliptical paraboloid with local minimum at $(-1, 2, 0)$
   (C) an ellipsoid with center at $(1, -2, 0)$
   (D) an ellipsoid with center at $(-1, 2, 0)$

5. The arc length of the curve given by $\mathbf{r}(t) = t\cos t\mathbf{i} + t\sin t\mathbf{j} + (2\sqrt{2}/3)t^{3/2}\mathbf{k}$, $0 \leq t \leq \pi$ is

   (A) $\pi + 1$
   (B) $\pi^2 + \pi/3$
   (C) $\pi^2 - 1$
   (D) $\pi^2/2 + \pi$

Short Answer: Please show all work.

6. (10 Points) Find parametric equations for the line in which the planes $x - 2y + 4z = 2$ and $x + y - 2z = 5$ intersect.

7. (20 points) A projectile is fired from the ground with an initial speed of 300m/s at an angle of elevation of $\alpha = 30^o$. For this problem let $g = 10$ m/s^2.

   (a) If $\mathbf{r}(t)$ is the position vector at any time t and satisfies $\frac{d^2\mathbf{r}}{dt} = -g\mathbf{j}$, find the expression for $\mathbf{r}(t)$.

   (b) When and how far away will the projectile strike? Do not quote formulas, please show from (a) how these quantities are derived.

   (c) How high will the projectile be when it is 3 km downrange? You need not simply your answer.

   (d) What is the greatest height reached by projectile? Please show from (a) how this quantity is derived.

8. (20 points) Consider the space curve $\mathbf{r}(t) = 6\sin(2t)\mathbf{i} + 6\cos(2t)\mathbf{j} + 5t\mathbf{k}$.

   (a) In words, describe what the quantities $\mathbf{T}$, $\mathbf{N}$ and κ mean.

   (b) Calculate $\mathbf{T}$, $\mathbf{N}$ and κ for the curve.

   (c) What, if any, geometric or mathematical relationship exists between $\mathbf{T}$, $\mathbf{N}$?