Math 107 Final Exam
December 19, 2018

Time: 2 hour and 25 minutes

Instructions: Show all work for full credit.
No outside materials or calculators allowed.

Extra Space: Use the backs of each sheet for extra space. Clearly label when doing so.

Name: ______________________________
ID #: ______________________________
Instructor/Section: __________________

“I pledge by my honor that I have abided by the NJIT Academic Integrity Code.”
_________________________ (Signature)

Formulas you may need for this exam:

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ A = P\left(1 + \frac{r}{n}\right)^n \]

\[ A = Pe^r \]

\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ b^2 = a^2 + c^2 - 2ac \cos B \]
\[ c^2 = a^2 + b^2 - 2ab \cos C \]

\[ \sin A = \frac{\sin B}{b} = \frac{\sin C}{c} \]

\[ A + B + C = 180 \]
1. Find the exact value. (8 points):
   
   a) \( \cot \frac{-11\pi}{2} \) 
   
   b) \( \csc \frac{2\pi}{3} \) 
   
   c) \( \sec \frac{7\pi}{6} \) 
   
   d) \( \cos \frac{5\pi}{3} \)
2. Solve the following equations. (12 points)
   a) \( b^2 - 14b = -7b + 8 \)

   \[
   b) \quad 2x^2 + 3x = 4x + 4
   \]

   \[
   c) \quad 7x^2 - 6x = 6x^2
   \]

   2a) ___________________

   2b) ___________________

   2c) ___________________

3. Let \( \log_a 2 = 1.2 \) and \( \log_a 3 = 1.5 \). Evaluate each of the following: (5 points)

   \[
   \begin{array}{|c|c|c|c|c|}
   \hline
   x & 4 & 3a & 6 & \sqrt{9} & 8a^2 \\
   \hline
   \log_a x & & & & & \\
   \hline
   \end{array}
   \]
4. Evaluate the function at the given value:
   (4 points)

   \[ f(a) = 2a^3 - 13a^2 + 2a + 30 \text{ at } a = 6 \]

5. (9 points)
   a) Expand the logarithm. \( \log_7(c\sqrt[3]{a}) \)

   5a) _____________________

   b) Condense the logarithm \( \frac{\log a}{3} + \frac{\log b}{3} + \frac{\log c}{3} \)

   5b) _____________________

   c) Convert to exponential form \( \log_9 76 = x \)

   5c) _____________________
6. (6 points) Find the equation of the line that satisfies the given conditions

   a. Through (-5,3); Perpendicular to \( y = 10x + 5 \)

   b. Through (2,-4); Parallel to the line \( y = -3x + 5 \)

6a. ________________________________ 6b. ________________________________

7. (6 points) Solve each equation

   a. \( 81^{3-2v} = 27^{-3v} \)

   a. ________________________________

   b. \( 3^{-3n-1} = 3^{2n+3} \)

   b. ________________________________

8. (6 points) Divide:

   \( (n^3 + 9n^2 + 6n - 19) \div (n + 2) \)

8) ________________________________
9. To approximate the length of a marsh a surveyor walks 11 yards from point A to point B, then turns 60° and walks 8 yards to point C. Approximate (round to the nearest tenth if necessary) the length of line AC of the marsh. Refer to the figure below. (10 points)

10. Graph one period of the function: \( y = 2\sin(3x) \). Identify the period and the amplitude. (4 points)

Period:_________________ Amplitude:_________________
11. Solve the following systems of equations. You may use any method. (4 points)
   a) \[
   \begin{align*}
   -2x + 8y &= -10 \\
   9x - 2y &= 11
   \end{align*}
   \]
   b) \[
   \begin{align*}
   3x - y &= -19 \\
   x + 7y &= 23
   \end{align*}
   \]

11a) ____________________________  11b) ____________________________

12. Sketch the graph of the function: \( y = 2\cos(x) + 1 \) Identify the period and amplitude. (4 points)

   Period: _______________  Amplitude: _______________
13. (5 points) Perform the indicated operation.

\[ h(t) = -4t - 1 \]

a. \[ g(t) = 3t + 1 \]
Find \((h + g)(t)\)

b. \[ f(x) = 2x \]

a.___________________________

\[ h(x) = 2x - 5 \]

b.___________________________

c. \[ g(x) = x^3 + 2x^2 \]
Find \(\left( \frac{h}{g} \right)(x)\)

c.___________________________

d. \[ h(t) = t^2 + 3t \]

d.___________________________

g(t) = 4t - 2
Find \((h + g)(-8)\)

e. \[ f(x) = 3x + 3 \]

\[ g(x) = x^2 - 1 \]

\[ f(x) = 3x + 3 \]
Find \((g - f)(2)\)

e.___________________________
14. Given the graph of $y = e^x$ below, use transformations to sketch the graph of: $y = e^{x^2} - 1$ (4 points)
15. Identify the vertex and axis of symmetry of each. Then sketch the graph. \( y = x^2 - 12x + 32 \) (6 points)

16. Give the degree and describe the end behavior of the graph of the polynomial function. 
\( f(x) = -x^3 + 14x^2 - 64x + 98 \) (3 points)

\[
\begin{align*}
x \to \infty, & \quad y = P(x) \to \quad \text{___________} \\
\end{align*}
\]

\[
\begin{align*}
x \to -\infty, & \quad y = P(x) \to \quad \text{___________} \\
\end{align*}
\]

16. Degree \______________________________
17. Evaluate the given logarithmic expressions. (4 points)

a) \( \log_7 343 = \) _____

c) \( \log_2 \frac{1}{16} = \) _____

b) \( \log_{49} \frac{1}{7} = \) _____

d) \( \log_5 \frac{1}{25} = \) _____