Math 110 Final Exam
December 16, 2019

Time: 2 hours and 30 minutes

Instructions: Show all work for full credit. No outside materials or calculators allowed.

Extra Space: Use the backs of each sheet for extra space. Clearly label when doing so.

Name: _______________________________
ID #: ___________________________
Instructor/Section: ____________________

“I pledge by my honor that I have abided by the NJIT Academic Integrity Code.”
_______________________________ (Signature)

Formulas are found on the last sheet of the exam; you may rip off that sheet for convenient use.
1. Solve the system of equations:

a. \[
\begin{align*}
&x^2 + y^2 = 9 \\
&y = x^2 - 3
\end{align*}
\]

b. \[
\begin{align*}
&2x + 3y - z = 13 \\
&5x - y + z = 0 \\
&x - 3y - z = -6
\end{align*}
\]
2. Simplify the following using partial fraction decomposition

a. \[
\frac{2x^3 - 4x^2 - x - 3}{x^2 - 2x - 3}
\]

b. \[
\frac{-2x + 4}{(x^2 + 1)(x + 1)}
\]
3. Evaluate the following limits:

a. \( \lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - x} \)

b. \( \lim_{x \to 0} \frac{\sqrt{x^2 + 100} - 10}{x^2} \)

c. \( \lim_{x \to 0} \sqrt{7 + \sec^2 x} \)
4. Consider \( f(x) = \frac{1}{x} \)

   a. Find a formula for the average rate of change of \( f(x) \)

   b. Take the limit of your result in part (a), as \( h \) goes to 0, to find the instantaneous rate of change for \( f(x) \).

   c. Evaluate your answer in part (b) at \( x = -2 \)
5. Below is the unit circle. For all angles shown (as well as the \textit{x-axis} and the \textit{y-axis}, label the following:

a. The angle measurements in degrees
b. The angle measurements in radians
c. The coordinates of the points on the circle.
6. Graph the following. Be sure to identify asymptotes and at least one identifying point. For trigonometric functions, graph at least 2 periods.

   a. \( y = e^{x-2} + 3 \)

   b. \( y = -4 \sin(3x + \pi) \)

   c. \( y = \ln(-x - 2) \)

   d. \( y = 1 - \sec(2x) \)
7. Solve the following trigonometric equations. If there are no solutions you must show all work proving this fact.

a. \( \sqrt{3}\sec x - 1 = 1 \) for all possible solutions

b. \( 1 - \sin^2 x - \cos x = 6 \) for all possible solutions

c. \( \cos \theta \sin \theta = \cos \theta \) for solutions in \([0, 2\pi)\)
8. Evaluate:
   
   a. \( sec^{-1}(\sqrt{2}) \)

   b. \( \csc\left(\arctan\left(-\frac{\sqrt{3}}{3}\right)\right) \)

   c. \( \tan\left(\sin^{-1}(1)\right) \)

9. Verify the following identity: \( \frac{\sec\theta}{\tan\theta} = \frac{\tan\theta}{\sec\theta - \cos\theta} \)
10. Given that \( \tan \theta = \frac{-1}{4} \) and \( \sin \theta > 0 \),

a. Find \( \sin 2 \theta \)

b. Find \( \cos 2 \theta \)

11. Solve the following:

a. \( \log x = 2 + \log(x - 1) \)

b. \( 8^{3x-2} = 9^{x+2} \)
Formulas:

Sum and Difference:
\[
\begin{align*}
\cos(u + v) &= \cos u \cos v - \sin u \sin v \\
\cos(u - v) &= \cos u \cos v + \sin u \sin v \\
\sin(u + v) &= \sin u \cos v + \cos u \sin v \\
\sin(u - v) &= \sin u \cos v - \cos u \sin v
\end{align*}
\]

Double

Standard Form of Equation of a Circle: General Form of Equation of a Circle
\[
(x - h)^2 + (y - k)^2 = r^2 \quad \quad \quad x^2 + y^2 + ax + by + c = 0
\]

Standard Form of Equation of an Ellipse: General Form of Equation of an Ellipse
\[
\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1. \quad \quad \quad Ax^2 + Cy^2 + Dx + Ey + F = 0
\]

Pythagorean Identity: \[\sin^2(x) + \cos^2(x) = 1\]

Polar to Rectangular Equations:
\[
r^2 = x^2 + y^2, \quad r \cos \theta = x, \quad r \sin \theta = y, \quad \tan \theta = \frac{y}{x}
\]

Double Angle:
\[
\sin 2 \theta = 2 \sin \theta \cos \theta, \quad \cos 2 \theta = \cos^2 \theta - \sin^2 \theta
\]
\[
\tan 2 \theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}, \quad \quad \quad \cos 2 \theta = 1 - 2 \sin^2 \theta
\]
\[
\cos 2 \theta = 2 \cos^2 \theta - 1
\]

Half Angle:
\[
\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}} \quad \quad \quad \cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}} \quad \quad \quad \tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}
\]

Average Rate of Change: \[\frac{f(x+h)-f(x)}{h}\]