MATH 337 — FINAL EXAM — SPRING 2018

Justify all your answers

1) (20 points) Let $A = [(3, -2, 0, -1, 0)^T, (2, 4, -1, 2, 0)^T, (0, 1, 0, 0, 0)^T, (1, 2, 1, -1, 0)^T, (3, 1, -5, 2, 2)^T]$
   
a) Compute $\det(A)$.
   
b) Compute $\det(2A^2A^T A^{-1})$

2) (20 points) Let $A = [(0, 1, 0, 0)^T, (0, 0, 1, 0)^T, (0, 0, 0, 1)^T, (1, 0, 0, 0)^T]$.
   
a) Find the inverse of $A$.
   
b) Find the inverse of $A^T A$.
   
c) Use part a) to solve $Ax = (1, 2, 0, 1)^T$. Is $Ax = b$ solvable for each $b \in \mathbb{R}^4$?

3) (20 points) Given $A = [(1, 0, 1)^T, (0, 1, 1)^T, (0, 0, 1)^T, (4, 5, 9)^T]$
   
a) Find bases and dimensions of $\text{Nul}(A)$, $\text{Col}(A)$ and $\text{Row}(A)$. What is the rank of $A$?
   
b) Is the system $Ax = b$ solvable for each $b \in \mathbb{R}^3$? Explain without finding an echelon form of $[A|b]$.

4) (20 points) Given $A = [(4, 0, 2)^T, (0, 4, 0)^T, (2, 0, 4)^T]$.
   
a) Find the eigenvalues and the corresponding eigenvectors. Show that the corresponding eigenspaces are orthogonal to each other.
   
b) Find a diagonalization of $A$. Do not compute the inverse present in this diagonalization.

5) (20 points) a) Show that the vectors $x_1 = (1, 1, 0)^T$, $x_2 = (1, 0, 1)^T$, $x_3 = (1, 0, 0)^T$ are linearly independent. Do they form a basis of $\mathbb{R}^3$? Explain.
   
b) Find an orthonormal basis of $\mathbb{R}^3$ using $x_1 = (1, 1, 0)^T$, $x_2 = (1, 0, 1)^T$ and $x_3 = (1, 0, 0)^T$.

6) (extra credit -10 points) Find the matrix corresponding to the quadratic form $Q(x) = x_1^2 + 2x_1 x_2 + 2x_1 x_3$. Is $Q$ positive definite, negative definite, or indefinite?