MATH 337 ——FINAL EXAM —-SPRING 2018

Justify all ypur answers

1) (20 points) Let $A = [(3,-2,0,-1,0)^T (2,4,-1,2,0)^T (0,1,0,0,0)^T (1,2,1,-1,0)^T (3,1,-5,2,2)^T]$

a) Compute det(A).

b) Compute $\det(2A^2A^TA^{-1})$

2) (20 points) Let $A = [(0,1,0,0)^T (0,0,1,0)^T (0,0,0,1)^T (1,0,0,0)^T]$.

a) Find the inverse of A.

b) Find the inverse of A^TA.

c) Use part a) to solve $Ax = (1,2,0,1)^T$. Is $Ax = b$ solvable for each $b \in R^4$?

3) (20 points) Given $A = [(1,0,1)^T (0,1,1)^T (0,0,1)^T (4,5,9)^T]$

a) Find bases and dimensions of Nul(A), Col(A) and Row(A). What is the rank of A?

b) Is the system $Ax = b$ solvable for each $b \in R^3$? Explain without finding an echelon form of$[A|b]$.

4) (20 points) Given $A = [(4,0,2)^T (0,4,0)^T (2,0,4)^T]$.

a) Find the eigenvalues and the corresponding eigenvectors. Show that the corresponding eigenspaces are orthogonal to each other.

b) Find a diagonalization of A. Do not compute the inverse present in this diagonalization.

5) (20 points) a) Show that the vectors $x_1 = (1,1,0)^T$, $x_2 = (1,0,1)^T$, $x_3 = (1,0,0)^T$ are linearly independent. Do they form a basis of R^3? Explain.

b) Find an orthonormal basis of R^3 using $x_1 = (1,1,0)^T$, $x_2 = (1,0,1)^T$ and $x_3 = (1,0,0)^T$.

6) (extra credit -10 points) Find the matrix corresponding to the quadratic form $Q(x) = x_1^2 + 2x_1x_2 + 2x_1x_3$. Is Q positive definite, negative definite, or indefinite?