Math 337 — Exam 1 — f2018

1) (20 points) Given the system

\[ \begin{align*}
3x_1 - 3x_2 + 3x_3 + 9x_4 &= b_1 \\
2x_1 - x_2 + 4x_3 + 7x_4 &= b_2 \\
3x_1 - 5x_2 - x_3 + 7x_4 &= b_3
\end{align*} \]

a) Write the system in the matrix form \( Ax = b \) with \( x = (x_1, x_2, x_3) \) and \( b = (b_1, b_2, b_3)^T \).

b) Is the system \( Ax = b \) solvable for each \( b \) in \( \mathbb{R}^3 \)?

c) Find the general solution of \( Ax = (3, 2, 3)^T \) in the form \( x = p + x_h \). Explain what \( p \) and \( x_h \) represent.

d) Does \( A \) have an inverse?

2) (20 points) Let \( T : \mathbb{R}^4 \rightarrow \mathbb{R}^3 \) be given by \( T(x_1, x_2, x_3, x_4) = (x_1 + x_2 - x_4, 2x_2 + x_3 + 4x_4, 3x_3 + 5x_4) \).

a) Find the standard matrix of \( T \).

b) Is \( T \) onto? one-to-one? Explain.

c) Are there any vectors \( x \) such that \( Tx = (2, -1, 3)^T \).

3) (20 points) a) Find the inverse of the matrix \( A = [(0, 2, -1)^T, (1, -2, 1)^T, (1, 0, 1)^T, (-1, -1, 1)^T] \)?

b) Are the columns of \( A \) linearly independent? Explain.

4) (15 points) Find an LU factorization of \( A = [(1, -1, 4, -2)^T, (3, -5, 2, -4)^T, (-5, 8, -5, 7)^T, (-3, 4, -7, 5)^T] \).

5) (15 points) Let \( A = LU \) with \( L = [(1, -2, 1, -2)^T, (0, 1, 2, -1)^T, (0, 0, 1, 0)^T, (0, 0, 0, 1)^T] \) and \( U = [(1, 0, 0, 0)^T, (3, -2, 0, 0)^T, (-5, 1, 0, 0)^T, (-3, 1, 0, 0)^T] \). Use the LU factorization of \( A \) to solve the system \( Ax = (1, 2, 9, -6)^T \).

6) (10 points) Let \( A \) be an \( m \) by \( n \) matrix and \( r \) be the number of its pivot columns. What are the conditions on \( m, n \) and \( r \) (other than \( r \leq m \) and \( r \leq n \) which is always true) such that the system \( Ax = b \)

a) has infinitely many solutions for each \( b \)?

b) has exactly one solution for each \( b \)?