Math 337 —-Exam 1—-f2018

1) (20 points) Given the system

$$3x_1 - 3x_2 + 3x_3 + 9x_4 = b_1$$

$$2x_1 - x_2 + 4x_3 + 7x_4 = b_2$$

$$3x_1 - 5x_2 - x_3 + 7x_4 = b_3$$

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a) Write the system in the matrix form Ax=b with $x = (x_1, x_2, x_3)$ and $b = (b_1, b_2, b_3)^T$.

b) Is the system Ax=b solvable for each b in R^3?

c) Find the general solution of $Ax = (3, 2, 3)^T$ in the form $x = p + x_h$. Explain what p and x_h represent.

d) Does A have an inverse?

2) (20 points) Let $T : R^4 \to R^3$ be given by $T(x_1, x_2, x_3, x_4) = (x_1 + x_2 - x_4, 2x_2 + x_3 + 4x_4, 3x_3 + 5x_4)$.

a) Find the standard matrix of T.

b) Is T onto? one-to-one? Explain.

c) Are there are any vectors x such that $Tx = (2, -1, 3)^T$.

3) (20 points) a) Find the inverse of the matrix $A = [(0, 2, -1)^T(1, -2, 1)^T(-1, -1, 1)^T]$?

b) Are the columns of A linearly independent? Explain.

4) (15 points) Find an LU factorization of $A = [(1, -1, 4, -2)^T(3, -5, 2, -4)^T(-5, 8, -5, 7)^T(-3, 4, -7, 5)^T]$.

5) (15 points) Let A=LU with $L = [(1, -2, 1, -2)^T(0, 1, 2, -1)^T(0, 0, 1, 0)^T(0, 0, 0, 1)^T]$ and $U = [(1, 0, 0, 0)^T(3, -2, 0, 0)^T(-5, 1, 0, 0)^T(-3, 1, 0, 0)^T]$. Use the LU factorization of A to solve the system $Ax = (1, 2, 9, -6)^T$.

6) (10 points) Let A be an m by n matrix and r be the number of its pivot columns. What are the conditions on m, n and r (other than $r \leq m$ and $r \leq n$ which is always true) such that the system Ax=b

a) has infinitely many solutions for each b?

b) has exactly one solution for each b?