

Math 337 ---Exam 1---s2018

1) (15 points) Let M be the augmented matrix of the linear system

$$x_1 + 2x_2 + 3x_3 = 0$$

$$2x_1 + 4x_2 + 4x_3 + 2x_4 = 2$$

$$3x_1 + 6x_2 + x_3 + 2x_4 = 14$$

- a) Find the reduced echelon form of M and identify its pivots.
 - b) Find the general solution of the system in the form $x = p + x_h$. Interpret p and x_h .
- 2) (20 points) Let $x_1 = (1, 1, -1)^T$, $x_2 = (2, 3, -4)^T$, $x_3 = (1, 0, 1)^T$ and $b = (b_1, b_2, b_3)^T$.
- a) Which vectors b are in the span $\{x_1, x_2, x_3\}$?
 - b) For which values of h are the vectors x_1, x_2, x_3 and $b = (h, h, h)^T$ linearly dependent?

3) (20 points) Let the standard matrix of a linear transformation T be $A = [(-1, 1, 2)^T(1, -2, -1)^T]$.

- a) Find T and its domain and codomain.
- b) Is T one-to-one? Explain.
- c) Is T onto? Explain.
- d) Are there any vectors x whose image under T is $(1, 1, 1)$?

4) (15 points) Let $T : R^2 \rightarrow R^2$ be the clockwise rotation by $\Pi/2$ and $S : R^2 \rightarrow R^2$ be the reflection through the line $x_2 = -x_1$.

- a) Find the standard matrices A and B of T and S respectively.
- b) Let $R=ST$ be the linear transformation obtained by first applying T and then S. Compute R and its standard matrix.

5) (20 points) a) Find the inverse of $A = [(2, 3, 0)^T(1, 2, -1)^T(4, 5, 1)^T]$?

b) Find the inverse of AA^T without computing first AA^T .

c) Is the system $Ax = b$ solvable for each b? If yes, find its solution.

6) (10 points) Let U be an echelon form of an $n \times n$ matrix A. Show that A is invertible if and only if all diagonal entries of U are nonzero.