Math 112 Exam #1

Sept. 27, 2017

Time: 1 hour and 10 minutes
Instructions: Show all work for full credit.
No outside materials or calculators allowed.
Extra Space: Use the backs of each sheet for extra space. Clearly label when doing so.

Name: ______________________

ID #: ______________________

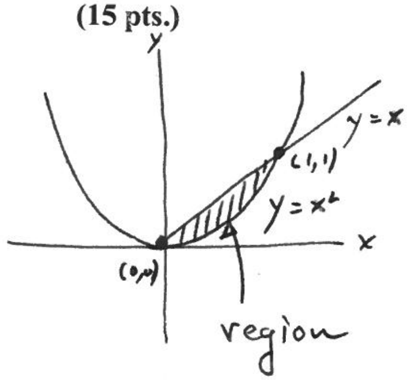
Instructor/Section: ______________

"I pledge by my honor that I have abided by the NJIT Academic Integrity Code."

______________________ (Signature)

Problem	Value	Score
1	15 pts.	
2	15 pts.	
3	20 pts.	
4	15 pts.	
5	15 pts.	
6	20 pts.	
TOTAL	100	

1. Find the volume generated by rotating about x-axis the region in the x,y- plane bounded below and above, respectively, by the curves $y = x^2$ and $y = x$, and sketch the region. **(15 pts.)**



$$V = \int_0^1 \pi x^2 - \pi x^4 \, dx$$

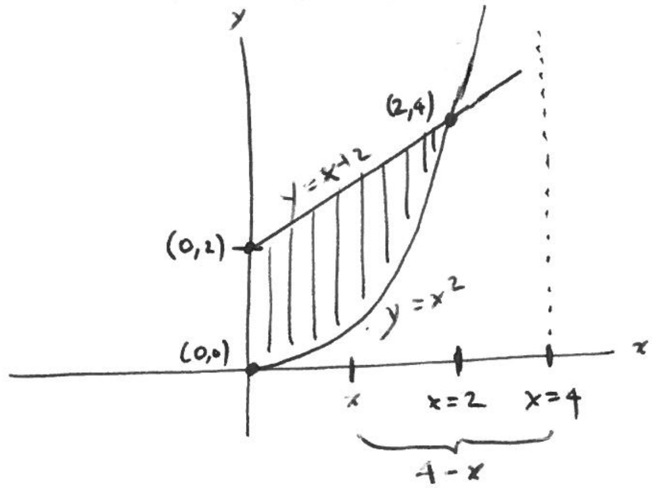
$$= \frac{1}{3}\pi x^3 - \frac{1}{5}\pi x^5 \Big|_0^1$$

$$= \frac{1}{3}\pi - \frac{1}{5}\pi$$

$$= \left(\frac{1}{3} - \frac{1}{5}\right)\pi$$

$$= \boxed{\frac{2\pi}{15}}$$

2. Let R in the x,y-plane be in the first quadrant and bounded by $y = x + 2$, $y = x^2$ and $x = 0$. Find the volume generated by revolving the region R about the line $x = 4$. **(15 pts.)**



$$V = \int_0^2 \left[(x+2) - x^2\right] 2\pi (4-x)\, dx$$

$$= 2\pi \int_0^2 (x^2 - x - 2)(x-4)\, dx$$

$$= 2\pi \int_0^2 (x^3 - 5x^2 + 2x + 8)\, dx$$

$$= 2\pi \int_0^2 \left[\frac{1}{4} x^4 - \frac{5}{3} x^3 + x^2 + 8x\right]_0^2$$

$$= 2\pi \left[\frac{1}{4} 16 - \frac{5}{3} \cdot 8 + 4 + 16\right]$$

$$= 2\pi \left[4 - \frac{40}{3} + 20\right]$$

$$= 2\pi \left[\frac{72}{3} - \frac{40}{3}\right]$$

$$= 2\pi \cdot \frac{32}{3}$$

$$= \boxed{\frac{64\pi}{3}}$$

3. Find the following: **(10 pts. each)**

(a) The arclength of the curve $y = \cosh x$, $0 \le x \le 1$, in the x,y-plane.

(b) The area of the surface generated by rotating the above curve around the x-axis.

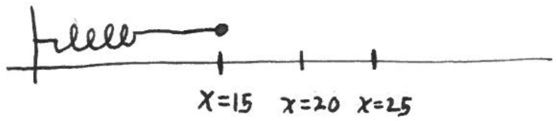
$$\text{(a)} \quad L = \int_0^1 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dx = \int_0^1 \sqrt{1+\sinh^2 x}\, dx = \int_0^1 \sqrt{\cosh^2 x}\, dx$$

$$= \int_0^1 \cosh x\, dx = \sinh x \Big|_0^1 = \boxed{\sinh(1) = \frac{e-e^{-1}}{2}}$$

$$\text{(b)} \quad A = \int_0^1 2\pi \cosh x\, dx = 2\pi \sinh x \Big|_0^1 = \boxed{\pi(e-e^{-1})}$$

$$= \boxed{2\pi \sinh(1)}$$

4. A spring with a natural length of 15 cm exerts a force of 45 Newtons when stretched to a length of 20 cm. Assuming Hooke's law, find the work done in stretching the spring from 20 to 25 cm. **(15 pts.)**



$$45 = K(20-15) \Rightarrow K = 9$$

$$\Rightarrow |F| = 9(x-15) \qquad (x > 15)$$

$$W = \int_{20}^{25} |F|\, dx = 9\int_{20}^{25} (x-15)\, dx = 9 \cdot \frac{1}{2}(x-15)^2 \Big|_{20}^{25}$$

$$= \frac{9}{2}\left[10^2 - 5^2\right] = \frac{9}{2} \cdot 75 = \boxed{\frac{675}{2} \text{ N}\cdot\text{cm}}$$

5. Find the derivatives of the following functions: **(5 pts. each)**

(a) $y=\sqrt{\sinh^2 x+1}$ (b) $y=\ln\left(\text{sech}(\theta+1)\right)$ (c) $y=4t^3\tanh\left(1/t^2\right)$.

(a) $y=\sqrt{\sinh^2 x+1} = \cosh x$

$y' = \dfrac{\frac{1}{2}}{\sqrt{\sinh^2 x+1}} \cdot 2 \cdot \sinh x \cdot \cosh x = \sinh x$

(same answer both ways)

$= \boxed{\sinh x.}$

(b) $y = \ln\left(\text{sech}(\theta+1)\right) = \ln\left(\dfrac{1}{\cosh(\theta+1)}\right) = -\ln\left(\cosh(\theta+1)\right)$

$\dfrac{dy}{d\theta} = -\dfrac{1}{\cosh(\theta+1)} \cdot \sinh(\theta+1) = \boxed{-\tanh(\theta+1)}$

(c) $y = 4t^3 \tanh\left(\frac{1}{t^2}\right)$

$\dfrac{dy}{dt} = 12t^2 \tanh\left(\frac{1}{t^2}\right) + 4t^3 \text{sech}^2\left(\frac{1}{t^2}\right)\cdot\left(\frac{-2}{t^3}\right)$

$= \boxed{12t^2\tanh\left(\frac{1}{t^2}\right) - 8\,\text{sech}^2\left(\frac{1}{t^2}\right)}$

6. Evaluate the following integrals: **(5 pts. each)**

(a) $\int\left(\left(\coth\sqrt{x}\right)/\sqrt{x}\right)dx$ (b) $\int_0^{\pi/6} 2\cosh(\sin\theta)\cos\theta d\theta$ (c) $\int \coth(3x)dx$

(d) $\int \sinh(\cosh x)\sinh x dx$

(a) $\int \frac{\coth\sqrt{x}}{\sqrt{x}}\,dx = 2\int \coth s\, ds = 2\ln(\sinh s) + C$

$= \boxed{2\ln(\sinh\sqrt{x}) + C}$

$s = \sqrt{x}$

$ds = \frac{1}{2}\frac{dx}{\sqrt{x}}$

(b) $\int_0^{\pi/6} 2\cosh(\sin\theta)\cos\theta\, d\theta \quad \int_0^{\frac{1}{2}} 2\cosh(u)\,du = 2\sinh(u)\Big|_0^{\frac{1}{2}}$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$= \boxed{2\sinh(\tfrac{1}{2})}$

$= \boxed{e^{\frac{1}{2}} - e^{-\frac{1}{2}}}$