Math 110 Common Exam #1
September 27, 2017

Time: 1 hour and 25 minutes

Instructions: Show all work for full credit. No outside materials or calculators allowed.

Extra Space: Use the backs of each sheet for extra space. Clearly label when doing so.

Name: _______________________________
ID #:   _______________________________
Instructor/Section: _________________

“I pledge by my honor that I have abided by the NJIT Academic Integrity Code.”
_______________________   (Signature)

Relevant Formulas for this Exam:

Circular motion and equations relating to a sector of a circle, radius r (as shown to the right).

\[ s = r\theta \]

\[ v = r\omega \]

\[ A = \frac{1}{2} r^2 \theta \]

\[ P = P_0 e^{kt} \]
1. (16 pts) Find the exact value of the following expression.
   a) \(2\ln \sqrt{e} + 2^{\log_2 4} + \log_2 [16]\)   
   b) \(\log_3 135 - \log_3 45\)
   c) \(\log_2 [e^{\ln 8}]\)
   d) \(2[\cos(45^\circ)]^2 + 2[\sin(45^\circ)]^2 - 2\tan(45^\circ)\)

2. (6 pts) Given that \(\cos \theta = \frac{2}{5}\), where \(\theta\) is an acute angle, find the exact values of the 5 other trigonometric functions. (Rationalize any and all denominators).
3. a) (4 pts) Write the expression in condensed (compressed) form with a coefficient of 1. (Assume all variables represent positive numbers)

\[ \frac{1}{3} \log(x + 2)^3 + \frac{1}{2} [\log(x^4) - \log(x^2 - x - 6)^2] \]

b) (4 pts) Write the expression in expanded form. (Assume all variables represent positive numbers)

\[ \log \sqrt{100x\sqrt{y}} \]

4. (10 pts) Graph the function \( y = -2^{x+2} - 1 \), on the set of axes below by making a table of values or using transformation. Be sure to label the asymptote on the graph, if any exists.
5. Given the 2 in. radius wheel and 7 in. radius wheel pulley system as shown below, find the following. *(You Can’t Use Ratios)*.

![Diagram of pulley system]

a) *(5 pts)* If the 2 in. radius wheel turns through an angle of 50°, what angle (expressed in degrees) does the 7 in. radius wheel turns through.

b) *(5 pts)* If the 2 in. radius wheel is spinning at a rate of 6 rpms, how many rpms is the 7 in. radius wheel making?
6. a) (5 pts) Find the height of a tree that casts a 100-feet shadow on the ground if \( \theta \) (the angle of elevation) to the sun from the ground is 45° as shown in the diagram below.

b) (5 pts) Given the right triangle as labeled below, if \( \sin \theta = \frac{1}{4} \), find the lengths of side ‘a’ and ‘c’. Simplify your answers as much as possible.
7. *(20 pts)* Solve the following equation for all real solutions. Make sure all answers are in the domain of the original problem.

a) \( \log_6(x + 2) + \log_6(x - 3) = 1 \)

b) \( 3xe^x + x^2e^x = 0 \)

c) \( 4(2^{6-2x}) - 5 = 27 \)

d) \( \log(x^2 + 1) = \log(x - 2) + \log(x + 3) \)
8. a) (5 pts) Suppose that \( \cos \theta = \frac{1}{x} \) where \( 'x' \) is a nonzero constant. Find the values of the other 5 trigonometric functions in terms of \( 'x' \). (*You do not need to rationalize the denominator*)

b) (5 pts) Given the rectangle (4 feet by 2 feet) inscribed in the semicircle as shown below, find the area of the shaded region of the semicircle.
9. a) (4 pts) Given that \( f(x) = 5^x + 5^{-x} \) and \( g(x) = 5^x - 5^{-x} \), evaluate the following expression below. Fully simplify your answer.
\[
[f(x)]^2 + [g(x)]^2
\]

b) (3 pts) Simplify completely (give your answer with positive exponents only)
\[
\left[\frac{(-x^2 y)^3 y^{-4}}{(xy)^5}\right]^{-2}
\]

c) (3 pts) Simplify completely.
\[
\frac{\sqrt{16 - 16x^2}}{4 - 4x}
\]